IRRIGATION MANAGEMENT DECISION MODEL USING PROBABLISTIC HYDROLOGIC AND IRRIGATION EFFICIENCY PARAMETERS

by

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ABSTRACT
A Bayesian decision theory optimization model was developed and applied for optimal irrigation management strategy. The model was used to select the optimum land area to be irrigated as controlled by stochastic hydrologic and probabilistic irrigation efficiency input parameters and the irrigator's own risk response function under the specified probabilistic conditions.

1. INTRODUCTION
Most developed models for solving complex irrigation system management problems applied steady state or purely deterministic input parameters; and so failed to consider the time and spatial variability's inherent in these factors. Moreover, the risk and uncertainty elements characteristically known to affect irrigation system planning, design and management processes were ignored. Hall and Buras [1], Connor et al. [2], Moore [3], Dudley et al.[4], Amir et al.[5], and delucia [6] studied stochastic water supply and demand and economic variability's using simulation, stochastic linear programming, and stochastic dynamic programming techniques. However, these studies that considered variable irrigation crop water use assumed a deterministic value for irrigation efficiency.

Irrigation efficiency is a critical, frequently used parameter in irrigation system design, planning and management. It is used for determining total irrigation water utilization and properly sizing irrigation system components. Although used extensively, the irrigation efficiency term is not often completely understood, as it is a complex function of many interacting and interdependent factors including irrigation system characteristics, management labour input, water rights and institutional factors [7]. The soil conditions, cropping patterns and irrigation practices, all varying in time and place, also influence irrigation efficiency. In reality, irrigation efficiency is a probabilistic phenomenon, and has not been adequately quantified. Its actual value is never reliably known at the time of design and management decisions. Frequently, a fixed value is assigned to it thus possibly leading to an oversized or undersized irrigation system component. Employing a fixed value for irrigation efficiency in modeling obviously would not yield an optimal irrigation management strategy as errors resulting from this practice would be greater than those from estimating evapotranspiration using climatic data, and some 'alien' formulas. Therefore, to develop an optimal irrigation management strategy, there is a need to conjunctively consider stochastic hydrologic events, probabilistic irrigation efficiency factor and risk response functions of irrigators frequently forced to make decisions under these specified unpredictable conditions. A Bayesian decision theory optimization model was developed and applied to select the optimum land area to be irrigated as constrained by these specified dynamic input factors. This model
was applied to the Wood River Valley Irrigation District No.45 located in Central Blaine County, Idaho [8].

Description of the Wood River Valley Irrigation District No.45:
The Wood River Valley Irrigation District No.45 lies entirely within the Bellevue Triangle; a mountain valley located in central Blaine County, Idaho. The district is approximately 3300 hectares. Three main earth canal systems totaling roughly 40 kilometers long divert irrigation water from a common point on the Big Wood River to irrigate about 3000 hectares of land. The Big Wood River has its source at the rugged mountains of the Saw tooth National Forest and a drainage area of about 1660 square kilometers.

The climate of the area is characterized by moderately cold winters and warm summers. The annual rainfall averages 380mm; 160mm of which occur during the months of December, January and February. The major irrigated crops are alfalfa, wheat, barley, oats and pasture.

The irrigation district was selected as the study area based on the following criteria:

a) availability on long-term basis of such relevant data as irrigation water diversions, crop consumptive use, and crop distribution;
b) availability of reliable current or updated economic data;
c) close relationship of the irrigation water diversion and the natural stream flow pattern;
d) history of persistent inadequate irrigation water availability during the late cropping season;
e) Willingness of the district managers to provide relevant information and records.

2. DECISION THEORY METHODOLOGY

Under probabilistic hydrologic and irrigation efficiency conditions, irrigation system design and management, decisions are definitely made under uncertainty. The Bayesian decision theory technique described and later applied is an efficient optimization tool for decision making because it utilizes both probabilistic and deterministic inputs [9,10] and is also equipped to refine and update prior limited knowledge as actual experimental data become available [11]. This feature that utilizes conditional probability theory and both subjective and objective inputs would allow irrigation management programs to incorporate the results generated from analyses of collected data.

A typical decision making process under probabilistic conditions can be outlined as follows [12]:

1. Choice action domain: \( A = \{a\} \). The decision maker wishes to select a single act, \( a \), from an, \( A \), domain of potential acts.

2. State, variable domain: \( \theta = \{0\} \). The decision maker is aware that the consequences of adopting an act, \( a \), depend on some state of nature which cannot be predicted with complete certainty. Each potential state of nature, \( 0 \), is embedded in a 9 domain.

3. Outcome domain: \( Z = \{z\} \). Each single act and a state of nature combination is associated with a potential terminal outcome, \( z \).

4. Utility evaluation: \( U(z, a, o) \). To be logically consistent with his basic preference among outcomes, the decision asker assigns a utility \( U(z, a, o) \) to the expected terminal outcome resulting from taking an action, \( a \), under a state of nature, \( 0 \).

Choice Action Domain

In a typical irrigation management decision process, an array of choice actions or decision variables exists. Under a set of unpredictable states of nature, each action will result in a matrix or stream of consequences or outcomes. The optimum land area, \( A_j \), to be irrigated considering water availability and irrigation system efficiency is a typical decision problem confronting the irrigators in the study area.
Therefore, the optimum land area to be irrigated \( A_j \), is the decision variable for the study model.

**State variable Domain**

The stream of expected consequences associated with a choice action is controlled by the existence of a set of state variables or states of nature the occurrence of which is unpredictable and therefore must be represented as probability density functions (pdf). Although many factors in irrigation systems management and design are in reality probabilistic only the hydrologic and the irrigation efficiency state variables are considered.

The total seasonal irrigation diversion for the Wood River Valley irrigation District No.45 largely exceeds the seasonal crop water requirement but is usually inadequate during the last half of irrigation season. Therefore the irrigation season was broken into two periods, P-1 (May to Mid-July to September) to correspond with the first cutting of hay which usually occurs in the second week of July. Also, after mid-July, river flows in the Big Wood River greatly decrease as there is no reservoir storage upstream of the point of diversion. By creating these periods, the impact on the decision strategy of the inadequate late season water supply could be adequately investigated. The relative frequency approach was used in generating the probability density function, pdf, for the continuous hydrologic state variable for each period (Tables 1 and 2).

A normal distribution was postulated for generating the probability density function for the irrigation efficiency state variable, as a frequency histogram of the irrigation efficiency data plotted for the district’s irrigation system closely resembles a normal distribution. Test of skewness and kurtosis on the computed efficiencies indicates that this is a good approximation. Moreover, Benjamin and Cornell [3] and Snedecor and Cockran [14] listed certain general conditions based on the central limit theorem; one of which must be satisfied to justify a normal distribution postulate. Since irrigation efficiency is a function of many factors including soil, crop, water rights, labour cost, management and others, that jointly and additively affect the parameter a normal distribution assumption is considered valid.

The pdf for the continuous irrigation efficiency state variable was determined once the population mean, \( \mu \), and the population standard deviation, \( \sigma \), were estimated from the 1975 and 1976 weekly irrigation efficiencies (Table 3).

**Table 1. Frequency Distribution of Irrigation Diversion for the Wood River Valley Irrigation District No. 45, for P-1 Period (May to Mid-July)**

<table>
<thead>
<tr>
<th>Flow intervals (x 10^{-6}m^3)</th>
<th>Hydrologic regime</th>
<th>Hydrologic state of nature ( \theta_i )</th>
<th>No. of years observed ( N_i )</th>
<th>Relative frequency of occurrence ( \frac{N_i}{\sum_{i=1}^{n} N_i} = (\theta_i) )</th>
<th>Cumulative relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0-12.0</td>
<td>Very poor</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12.0-24.0</td>
<td>Poor</td>
<td>( \theta_2 )</td>
<td>4</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>24.0-36.0</td>
<td>Inadequate</td>
<td>( \theta_3 )</td>
<td>10</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>36.0-48.0</td>
<td>Marginal</td>
<td>( \theta_4 )</td>
<td>12</td>
<td>0.21</td>
<td>0.46</td>
</tr>
<tr>
<td>48.0-60.0</td>
<td>Fairly Adequate</td>
<td>( \theta_5 )</td>
<td>23</td>
<td>0.41</td>
<td>0.87</td>
</tr>
<tr>
<td>60.0-72.0</td>
<td>Adequate</td>
<td>( \theta_6 )</td>
<td>7</td>
<td>0.13</td>
<td>1.00</td>
</tr>
<tr>
<td>72.0-84.0</td>
<td>Good</td>
<td>( \theta_7 )</td>
<td>0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Frequency distribution of irrigation diversion for the wood river valley irrigating district no. 45, for P-2 period (mid-July to September)

<table>
<thead>
<tr>
<th>Flow intervals (x10^6 m^3)</th>
<th>Hydrologic regime</th>
<th>Hydrologic state of nature</th>
<th>No of years observed</th>
<th>Relative frequency ( N_i / \sum N_i = \theta_i )</th>
<th>Cumulative relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0-12.0</td>
<td>Very poor</td>
<td>( \theta_1 )</td>
<td>12</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>12.0-24.0</td>
<td>Poor</td>
<td>( \theta_2 )</td>
<td>15</td>
<td>0.27</td>
<td>0.48</td>
</tr>
<tr>
<td>24.0-36.0</td>
<td>Inadequate</td>
<td>( \theta_3 )</td>
<td>18</td>
<td>0.32</td>
<td>0.80</td>
</tr>
<tr>
<td>36.0-48</td>
<td>Marginal</td>
<td>( \theta_4 )</td>
<td>11</td>
<td>0.20</td>
<td>1.00</td>
</tr>
<tr>
<td>48.0-60.0</td>
<td>Fairly Adequate</td>
<td>( \theta_5 )</td>
<td>0</td>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>60.0-72</td>
<td>Adequate</td>
<td>( \theta_6 )</td>
<td>0</td>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>72.0-84.0</td>
<td>Good</td>
<td>( \theta_7 )</td>
<td>0</td>
<td>0.0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ \sum_{i=1}^{n} N_i = 56 \sum_{i=1}^{n} (\theta_i) = 1.00 \]

Table 3. The Irrigation Efficiency Probability Density Function for the Wood River Valley Irrigation District No. 45 (Based on a mean, \( \mu \), of 0.20 and a standard deviation, \( \sigma \), of 0.07).

<table>
<thead>
<tr>
<th>Irrigation efficiency range</th>
<th>Irrigation efficiency regime</th>
<th>State of nature (( \theta_k ))</th>
<th>Prior probability of occurrence ( p(\theta_k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0-0.10</td>
<td>Very low</td>
<td>( \theta_1 )</td>
<td>0.08</td>
</tr>
<tr>
<td>0.10-0.15</td>
<td>Low</td>
<td>( \theta_2 )</td>
<td>0.16</td>
</tr>
<tr>
<td>0.15-0.20</td>
<td>Moderately low</td>
<td>( \theta_3 )</td>
<td>0.26</td>
</tr>
<tr>
<td>0.20-0.25</td>
<td>Marginal</td>
<td>( \theta_4 )</td>
<td>0.26</td>
</tr>
<tr>
<td>0.25-0.3-0</td>
<td>Fair</td>
<td>( \theta_5 )</td>
<td>0.16</td>
</tr>
<tr>
<td>0.30-0.35</td>
<td>Moderately high</td>
<td>( \theta_6 )</td>
<td>0.06</td>
</tr>
<tr>
<td>0.35-0.40</td>
<td>High</td>
<td>( \theta_7 )</td>
<td>0.02</td>
</tr>
<tr>
<td>0.40-0.45</td>
<td>Very high</td>
<td>( \theta_8 )</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\[ \int_{-\infty}^{\infty} f(\theta) \, d\theta = 1.00 \]

Outcome Domain and Utility Evaluation
A decision variable and a set of state variables combination yields a consequence matrix that must be transformed to a scale structure incorporating the risk behavior tendency of irrigators confronted with decision making under specified uncertain state variables. Proper transformation of the consequence matrix would necessitate generating and using the irrigators’ own utility functions and maximizing the expected monetary value, EMV, or the expected utility, EU, rather than a purely economic objective.
function [15]. Utility is a measure of the intrinsic worth of a particular outcome to an irrigator. Since utility functions are transient [16], two functions were generated for the irrigators in the study area using the established von Neumann and Morgenstern [17] model approach. If irrigation management involves a series of frequent repeated decision making processes, as in fact it does, and monetary values adequately reflect payoffs, a linear utility function in figure 1 would become appropriate [18]. A curvilinear utility function as generated depicts the risk aversion tendency of some interview irrigators (fig.2).

The transformed outcome matrix is summed up over all the state variables for one decision variable. The decision variable yielding the maximum total expected utilities is the optimum strategy.

Fig. 1. Linear utility function of irrigators in the Wood River Valley Irrigation District No.45.

The transformed outcome matrix is summed up over all the state variables for one decision variable. The decision variable yielding the maximum total expected utilities is the optimum strategy.

Fig. 2. Utility function of Irrigators in the Wood Valley Irrigation District No.45.

3. MODEL DEVELOPMENT

The crop water response, consumptive use, irrigation crop production cost and crop income are essential input functions that must be generated prior to model formation.

CROP RESPONSE

The crop response function relates specific crop response to varying amounts of applied irrigation water, and in modelling it creates a link between the hydrologic and economic components [19]. Development of a unique reliable crop response function would require intensive research as many interacting and interdependent factors influence crop response. Jensen [7] suggested the use of both linear and curvilinear crop functions under certain soil moisture stress and soil moisture availability conditions.

The relationship between the expected crop yield, the maximum crop yield and irrigation amount can be represented by the following equations:

\[ Y = \lambda^1 Y_{\text{max}} \]  \hspace{1cm} (1)
\[ \lambda^1 = f(\lambda) \]  \hspace{1cm} (2)
\[ \lambda = \frac{Q \eta}{CV A} \]  \hspace{1cm} (3)

0.2 < \lambda \leq 1 \hspace{1cm} (4)
0 \leq \lambda^1 \leq 1 \hspace{1cm} (5)

for \( \lambda^1 = 1 \) and \( \lambda = 1 \), \( Y = Y_{\text{max}} \) \hspace{1cm} (6)

where
Y = crop yield in units per ha dependent upon the water supply,
\[ Y_{\text{max}} = \text{maximum crop yield under optimum water supply,} \]
\[ \lambda = \text{crop response coefficient} \]
\[ n = \text{overall irrigation system efficiency expressed as decimal} \]
CU = seasonal consumptive use in m³ for the model crop or combination of crops
A = irrigated area in ha
\[ \lambda^2 = \text{crop response function shape factor} \]
Q = seasonal irrigation diversion

Both the linear and polynomial crop response functions applied are shown in figs.3 and 4. The crop yield is related to the maximum crop yield by the water supply regime and the shape of the crop response function by equations (1) into (6)

\[ \text{Fig.3. linear type crop response function} \]

**Consumptive Use**

Jensen and Wright [20] determined the weekly consumptive use, CU for 1975 for the major irrigated crops in the area. The 1976 CU was computed using climate data and the Penman equation. The weighted seasonal CU for crops grown in the irrigation district (60 percent alfalfa and 40 percent wheat) was 0.4m for P-1 and 0.3m for P-2.

**Crop Production Costs**

The multi-crop production cost input shown in Table 4 represents a weighted cost based on a crop distribution of 60% alfalfa and 40% wheat. In addition, a charge for irrigation water of 0.15 kobo per m³ was assumed as this value compares quite closely with district water assessment figures.

**Table 4. Irrigation crop production costs for combined wheat and alfalfa in Naira per ha per period.**

<table>
<thead>
<tr>
<th>Cost category</th>
<th>Crop (Alpha/alpha / wheat)</th>
<th>Operating inputs</th>
<th>Capital cost</th>
<th>Ownership cost</th>
<th>Labour cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Operating inputs</td>
<td>Capital cost</td>
<td>Ownership cost</td>
<td>Labour cost</td>
<td>Total Cost</td>
</tr>
<tr>
<td>Operating inputs</td>
<td>42.90</td>
<td>8.10</td>
<td>10.00</td>
<td>9.50</td>
<td>70.50</td>
<td></td>
</tr>
<tr>
<td>Capital cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ownership cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labour cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>70.50</td>
</tr>
</tbody>
</table>

**Crop Income:**

With adequate water supply, alfalfa yield averages 10 tons per ha in 2 cuttings, while grain yield averages 110 bushels per ha. The 1976 and 1977 crop prices per ton of alfalfa and per bushel of wheat were #30 and #2.30 respectively.

As the model is designed to make an optimal irrigation decision at the end of each period, P-1 and P-2, an action-state monetary payoff matrix
must be generated during each period. Since wheat is not normally harvested in mid-July, and some monetary function must be developed for the period p-1, a technique used by Conklin and Schmisseur [21] was modified and used for determining what proportion of the total seasonal harvest or yield could be attributed to each of the periods p-1 and p-2. The modified version becomes:

\[ Y = (\text{Growth}_p) (\lambda^1_p) (Y_{\text{max}}) \]  (7)

Where; p = period
Growth = yield coefficient per period, P, for a crop.

The seasonal crop yield coefficients for wheat developed by Conklin and Schmisseur [21] were used; which were 0.9 and 0.1 for periods p-1 and p-2, respectively. Alfalfa was unaffected as one cutting was produced during each period.

4. MODEL FORMULATION
Conceptual model: Deterministic Approach
When the decision is to irrigate just one hectare of land under a single crop, j, the germinal monetary payoff function for this action can be formulated as follows:

\[ \Pi_j = \Pi_c Y - \Pi_w q - \Pi_p CP - \Pi_w Q - \Pi_m \]  (8)

where
\( \Pi_j \) = net farm income in naira for the decision to irrigate one ha under a single crop, j.
\( \Pi_c \) = unit price in naira received from harvested crop
\( Y \) = crop yield in units per ha dependent upon the water supply
\( \Pi_w \) = per m$^3$ water assessment ln naira
\( q \) = applied irrigation water in m$^3$ per ha
\( \Pi_p \) = per ha irrigation crop production operating cost in naira
\( \Pi_m \) = per ha irrigation crop production capital cost (interest on capital investment) in naira
\( \Pi_m \) = per ha irrigation crop production ownership cost (depreciation, taxes, and insurance) in naira
\( \Pi_l \) = per ha irrigation crop production labour cost in naira

However, if the decision is to irrigate, \( A_j \), ha of land under a single crop, j, then the total monetary payoff function becomes:

\[ \Pi_{A_j} = (\Pi + \Pi_w q) A_j - \Pi_w Q \]  (9)

where
\( A_j \) = The total monetary payoff for irrigating \( A_j \) ha under a single crop, j
\( Q \) = irrigation diversion in m$^3$

Max: \( z = \Pi_{A_j} \) (10)
Subject to the boundary conditions established by the crop response functions as follows:
1. \( \lambda = 1 \), implies a sufficient irrigation water supply with \( Y = Y_{\text{max}} \)
2. \( 0.2 < \lambda < 1 \), implies a range of non-optimal water supply with \( Y = \lambda Y_{\text{max}} \)
3. \( 0.0 < \lambda < 0.2 \) = a range of extreme moisture stress with \( Y = 0 \).

Optimality is dependent upon the computation of that area, \( A_j \), that maximizes the objective, function, \( \Pi_{A_j} \), subject to the specified constraints:

Decision Theory Model Formulation
The objective function of the decision theory stochastic optimization scheme can be formulated on the basis of the expected monetary value, EMV criterion or the expected utility, EU, criterion. The EMV criteria computed using the linear utility
function (Figure 1) and the EU criterion utilizes the generated curvilinear utility function (Figure 2).

Decision Theory Model Formulation: EMV Criterion
By deciding to irrigate, \( A, \) hectares of land under a single specified crop, \( j, \) when the probability, \( P(\theta_i), \) of the occurrence of the state variable, \( \theta_i, \) for stream flow and the probability, \( p(\theta_k), \) of the occurrence of the state variable, \( \theta_k, \) for irrigation efficiency are considered, the total expected monetary payoff function, EMP, can be expressed as follows:

\[
EMP_{ijk} = m \sum_{k=1}^{m} \sum_{i=1}^{n} [p(\theta_i) \Pi_{ijk}]
\]

where:

\( \Pi_{ijk} \) = the terminal monetary payoff function for deciding to irrigate, \( A, \) hectares of land, under a single crop, \( j, \) when \( i \) and \( k \) are the states of nature considered.

\( P_{ijk} = (P_c Y_{ik} - OP - CP - OW - LA) A_j - P_w Q_{ik} \) (12)

\( P(\theta_k) = \) the probability of the occurrence of the state variable, \( \theta_k \)

\( P(\theta_i) = \) the probability of the occurrence of the state variable, \( \theta_i \)

\( n = \) number of the considered hydrologic states of nature

\( m = \) number of the considered irrigation efficiency states of nature

\( A_j = \) area in ha under crop

\( Q_{ik} = \) seasonal irrigation diversion available when \( i \) and \( k \) are the states of nature considered

\( Y_{ik} = \) crop yield in units per ha when \( i \) and \( k \) are the states of nature considered

For the EMV criterion, the objective function can be expressed as:

\[
\text{Max } Z = EMP_{ijk} \quad (13)
\]

subject to:

\( y_{ik} = \lambda^i k Y_{max} \)

The decision trees shown in figs. 5 and 6 schematically illustrate the decision processes for the EMV criterion. Since \( \theta_i \) and \( \theta_k \) are considered stochastic independent; the tree of fig. 5 can be reduced to that of fig. 6.
Decision Theory Model For Formulation: EU Criterion

Similarly, if the decision is to irrigate, $A_i$, hectares of land under a single crop, $j$, when the probability, $p(\theta_i)$, of the occurrence of the state variable, $\theta_i$, for stream flow and the probability, $p(\theta_k)$, of the occurrence of the state variable, $\theta_k$, for irrigation efficiency are considered, the total expected utility function can be expressed as:

$$EU_{ijk} = \sum_{k=1}^{m} \sum_{i=1}^{n} [p(\theta_i)(\theta_k)U_{ijk}]$$  (15)

Where

$U_{ijk} = \alpha \Pi_{ijk}$

$\alpha \Pi_{ijk}$ implies that the terminal monetary off function, $\Pi_{ijk}$, must be first transformed to their equivalent utility functions, $u_{ijk}$ such as illustrated in fig. 2. The other parameters are previously defined.

For the EU criterion, the objective function can be mathematically expressed as follows:

Max $z = EU_{ijk}$  (16)

Similarly, subject to the boundary conditions expressed in eqn. (14). The decision tree in fig. 7 schematically illustrates the decision processes for EU criterion.

Determination of the optimal irrigation management strategy involves a sequential search through various input areas, $A_j$, and locating that area that area that yields the greatest outcome or consequence matrix. Thus, the optimal strategy is dependent upon and only upon that area, $A_j$; yielding the greatest EMV or EU. That is, $A_j$ is selected if and only if the following conditions are satisfied:

$E[\text{MV}(A_j, \theta_{ik})] > E[\text{MV}(A_w, \theta_{ik})]$ for all $w$  (17)

$E[\text{U}(A_j, \theta_{ik})] > E[\text{U}(A_w, \theta_{ik})]$ for all $w$  (18)

The optimality search procedure adopted is illustrated by a simplified flow diagram (fig. 8). Listings of the computer program that is equipped with a graph plotter routine are available.

6. MODEL TESTING

The following cases were established in testing the developed model:

1. Prior probabilistic case that considered both prior probabilistic hydrologic and irrigation efficiency functions. The prior pdf for the irrigation efficiency factor was derived by postulating a normal distribution.

2. Posterior probabilistic case that considered both probabilistic hydrologic and irrigation efficiency factors. However, the prior pdf for the irrigation efficiency was transformed to a posterior pdf using Baye's theorem. This transformation was essential because an experiment in 1976 showed that the actual irrigation efficiency for the district's irrigation system was 20 percent [8].

Bayesian strategy permits making the assumptions necessary for generating the prior pdf for the irrigation efficiency state variables and is also equipped to update or revise the prior pdf as new data become available.
Fig. 8  Simplified Flow Diagram of the Sequential Optimality Search Procedure
The posterior pdf input into the model was derived elsewhere [8]. But the general form of the Baye’s theorem that was applied can be written as follows:

\[ P(\theta_k|Z) = \frac{P(Z|\theta_k)P(\theta_k)}{\sum_{i=1}^{n} P(Z|\theta_i)P(\theta_i)} \]  

(19)

That is:

\[ p_{state}^{sample} = \frac{p_{state}^{sample}p_{state}}{\sum_{state} p_{state}^{sample}p_{state}} \]  

(20)

in which
\( \theta_k \) = unknown state of nature
\( Z \) = observed sample
\( n \) = all considered states

7. RESULTS AND DISCUSSION

In case i, the maximum land areas to be irrigated for the different periods and the associated maximum EMV and the maximum EU are listed in Tables 5 and 6. A polynomial type crop response function resulted in greater areas, greater maximum EMV and maximum EU than a linear type response function. The reason is that the area under the polynomial function is larger than the area under the linear crop function. Negative EMV values were obtained under a linear type crop response during P-2, due to the impact of the seasonal crop yield coefficients for wheat and the limited irrigation water availability during P-2. It would therefore be a poor management decision to irrigate wheat during P-2. The plot in Figs. 9 and 10 show the relationships between the expected utilities, EU and the associated choice actions. The peak of the curve defines the optimal point as specified in the equation 17 or 18.

Table 5: Maximum land area to irrigate at different periods under a normal distribution assumption for irrigation efficiency: Expected monetary value, EMV, criterion.

<table>
<thead>
<tr>
<th>Crop Response Function</th>
<th>Irrig. Period, P-1</th>
<th>Irrig. Period, P-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Max. EMV: 15.00 N\times 10^{-4}</td>
<td>Max. Area: 2400</td>
</tr>
<tr>
<td>Polynomial</td>
<td>Max. EMV: 22.00 N\times 10^{-4}</td>
<td>Max. Area: 3200</td>
</tr>
</tbody>
</table>

Table 6. Maximum land area to irrigate at different periods under a normal distribution assumption for irrigation efficiency: Expected utility, EU, criterion.

<table>
<thead>
<tr>
<th>Crop Response Function</th>
<th>Irrig. Period, P-1</th>
<th>Irrig. Period P-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Max. EU: 90.50</td>
<td>Max. Area: 2400</td>
</tr>
<tr>
<td>Polynomial</td>
<td>Max. EU: 91.88</td>
<td>Max. Area: 2800</td>
</tr>
</tbody>
</table>

By incorporating the posterior pdf into the model and using a polynomial type crop function, an improved irrigation management decision resulted, conforming closely to the normal irrigation management practices obtaining in the district (Table 7 and 8). In the district, about 3000 hectares of land are usually irrigated in the period P-1, and this area is later cutback to about 2000 hectares in the period P-2.

In other post optimal analyses, not included in this paper, it was found that the developed model was sensitive to and independent upon the crop response functions, the decision objective criteria, the probability density function for irrigation efficiency variants, and the seasonal crop yield coefficients. The
irrigation multi-crop production cost factors and the irrigation water use assessments did not impact the optimal management strategy as

Fig. 9: Expected utility – irrigated area curve, for period p-1: a polynomial crop response function for a multi-crop system.

Fig 10. Expected Utility - Irrigated Area Curve, for Period P-2: a Polynomial crop Response Function for a Multi-crop System.
much as the unreliable hydrologic, highly probabilistic irrigation efficiency data and variable seasonal crop yield factors [8].

Table 7: Optimal Multi-crop Land Area to irrigate when 20% irrigation efficiency was measured: Expected monetary value, EMV, Criterion

<table>
<thead>
<tr>
<th>Crop Response Function</th>
<th>Irrig. Period P-1</th>
<th>Irrig. Period P-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max EMV:</td>
<td>Optimal</td>
</tr>
<tr>
<td>Linear</td>
<td>18.00</td>
<td>2800</td>
</tr>
<tr>
<td>Poly-Nominal</td>
<td>26.00</td>
<td>3600</td>
</tr>
</tbody>
</table>

Table 8: Optimal Multi-crop Acreages, to irrigate when 20% irrigation efficiency was measured: Expected Utility, criterion

<table>
<thead>
<tr>
<th>Crop Response Function</th>
<th>Irrig. Period P-1</th>
<th>Irrig. Period P-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max EU:</td>
<td>Optimal</td>
</tr>
<tr>
<td>Linear</td>
<td>91.19</td>
<td>2800</td>
</tr>
<tr>
<td>Polynomial</td>
<td>92.75</td>
<td>3200</td>
</tr>
</tbody>
</table>

8. PROBABLE SOURCES OF ERROR AND RECOMMENDATIONS

The developed Bayesian decision theory model would definitely require some refinement to increase its usefulness and reliability due to the following unrealistic assumptions:

i) The crop response function. The two crop response functions used in the model were synthetic. That is, they were not developed from research. The functions neither provided a penalty function for over irrigation causing irrigation water wastages, leaching and possibly drainage problems, nor for under irrigation possibly leading to adverse soil moisture stresses.

ii) The utility function. The utility function applied was the average function for the manager and two members of the board of directors of the irrigation district. A realistic model should incorporate an average function for some of the irrigators themselves. Even the function used was highly dynamic as another attempter generate it resulted in highly erratic data. This is understandable, as human minds are generally highly inconsistent particularly when preferences among outcomes differ slightly.

iii) Stochastic independence of the state variables. The stochastic independence of the hydrologic and irrigation efficiency variables should be investigated. It does appear that a conditional relationship or dependence exists between them.

iv) Irrigation crop production cost function. The irrigation cost components applied were those for the Twin Falls and Jerome Counties instead of the Blaine county where the study was conducted. Brockway (1978) confirmed that significant cost variations exist in the counties.

9. CONCLUSION

The developed Bayesian decision theory model is relatively simple and flexible. It utilizes purely subjective data inputs to generate an irrigation management decision. This feature makes it applicable in the developing nations where arbitrary estimates must be made in the face of inadequate or scarce data.

In the developed nations where research is strong in irrigation technology, the model can also be used for constantly updating the subjective data using the new data from research, to generate irrigation management policy

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5. Amir, I., Y. Friedman, S. Sharon and A. Ben-David, A combined model for operating, irrigated agricultural systems under uncertainties, Transactions of the ASAE, 19(2), 299-304,1976.


