RELIABILITY-BASED ANALYSIS OF ALUMINIUM LAMINATED SOLID TIMBER COLUMNS USING SELECTED NIGERIAN TIMBER SPECIES

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ABSTRACT
This paper presents the results of safety assessment of timber columns laminated with aluminium using the First Order Reliability Methods. Three failure modes were considered in the studies: bending failure, buckling failure, and flexural buckling failure modes. The results show that the column is safer for compression failure mode which has safety index values of 4.3 and 9.68 for imposed load and dead-to-live load parameters respectively without laminates, and progressive safety indices of up to 12.4 and 20.87 respectively for columns with laminates of 20mm thickness. The study showed that the most critical failure mode for the column is the flexural buckling mode. It is therefore deduced that from the results of the critical failure mode, laminate thickness of 10mm should be used to withstand any variation in load ratios of 0.6, 0.8 and 1.0 and a maximum imposed load of 15kN be used to ensure a safe column design.

Keywords: reliability, critical failure mode, stochastic, composite, limit state

1. INTRODUCTION
The analysis and design of timber structures are done bearing in mind the fact that timber is of variable strengths both within same species and for various species. These variations are accounted for by the natural growth of their parent trees whereby there is little or nothing that can be done to control the final strength properties (physical and mechanical) of the resulting structural timber. The use of timber for structural purposes cuts on the emission of greenhouse gases which are products of the manufacture of conventional building materials. The good aspect of timber is that it has a very high strength-to-weight ratio, it is capable of transferring both tension and compression forces, and is naturally suitable as a flexural member [1]. Composite engineering seeks ways to overcome the limitations present in both constituent materials by using the more pronounced strength properties on one material to cover up for the defects or limitations found in the other material. Lamination is a technique of external reinforcement of structural components. The limitation in sectional properties of available timber is also a factor for the use of Aluminium laminates. Aluminium is a lightweight and durable metal. Aluminium has two main advantages when compared with other metals for structural use. Firstly, it has a low density, about one third that of iron and copper. Secondly, although it reacts rapidly with the oxygen in air, it forms a thin tough and impervious oxide layer which resists further oxidation [2]. This makes aluminium desirable in structures especially in marine areas. The main mechanical properties of Aluminium are: elastic limit (f₀₂) or yield stress (fᵧ), ultimate strength (fₚ), Young’s modulus (E): 70,000N/mm², ultimate elongation (ε), specific weight (γ): 27,000N/m³, thermal elongation coefficient (α): 23x10⁻⁶ per °C and Poisson Ratio (ν): 0.3 [3, 4]. Reliability of structural systems can be defined as the probability that a structure under consideration has a proper performance throughout its lifetime [5]. The aim of structural reliability assessment is to quantify the reliability of structures under consideration of all uncertainties associated with the formulation of the failure criteria of the structure [6, 7]. Reliability...
methods are used to estimate the probability of failure. The reliability, estimated as a measure of the safety of a structure, can be used in a decision (design) process.

The aim of this work is to carry out reliability-based evaluation of the structural performance of composite solid slender timber columns with Aluminium laminates using selected Nigerian timber species. It is based on the modification of the strength of solid timber columns with the introduction of Aluminium laminates. The strength variation of the timber is assessed to determine the strength behaviour of timber columns with varying Aluminium laminate thickness. The selected Nigerian timbers to be used in this study are Strombosia pustulata, Macrocarpa bequeriti, Nauclea diderrichii and Entandrophragma cylindricum which have local names of Itako, Oporoporo, Opepe and Ijebu respectively [8].

Reliability analyses of solid timber columns laminated with aluminium sheet of varying thickness were carried out. The reliability processes considered three failure modes which are bending, buckling and flexure. MATLAB [9] was used to run the First Order Reliability Method analyses incorporating programs that were designed for the three failure modes.

2. LOAD MODELS ON A COMPOSITE COLUMN

2.1 Heterogeneous bars under direct stress (compression failure mode)

For the composite timber-aluminium column that is subjected to compression stresses, the materials will be strained by equal amounts. The timber of cross sectional area, \( A_T \), and young’s modulus \( E_T \), the resulting stress being \( f_T \), and the aluminium having corresponding values of \( A_A \), \( E_A \) and \( f_A \). If the composite column is under a load, \( P \), the initial strain, \( x \), is given as

\[
x = \frac{f_A}{E_A}
\]

And the total load is given as

\[
P = A_A f_A + A_T f_T
\]

\( A_A f_A \) and \( A_T f_T \), being the loads carried by each of the constituent material.

\[
f_A = E_A x = \frac{E_A}{E_T} f_T
\]

Therefore,

\[
P = f_T \left( A_T + \frac{A_A E_A}{E_T} \right)
\]

Hence the stress in the timber section will be

\[
f_T = \frac{P}{A_T \left( 1 + \frac{A_A E_A}{A_T E_T} \right)}
\]

Conversely, the stress in the aluminium section will be

\[
f_A = \frac{P}{A_A \left( 1 + \frac{A_T E_T}{A_A E_A} \right)}
\]

And the total compressive stress acting on the column will be

\[
P = A_T \left( 1 + \frac{A_A E_A}{A_T E_T} \right) + A_A \left( 1 + \frac{A_T E_T}{A_A E_A} \right)
\]

From the performance function \( G(x) = R - S \),

\[
G(x) = k_{mod} f_{c,0,k} + \alpha f_{c,0,k}
\]

\[
- Q_k \left( \gamma_g \alpha + \gamma_q \right) \left( \frac{1}{A_T \left( 1 + \frac{A_A E_A}{A_T E_T} \right)} \right)
\]

\[
+ \frac{1}{A_A \left( 1 + \frac{A_T E_T}{A_A E_A} \right)}
\]

Where \( k_{mod} \) is the modification factor for duration of load and moisture content, \( f_{c,0,k} \) is characteristic compressive strength parallel to grain, \( \alpha \) is load ratio, \( \gamma_g \) and \( \gamma_q \) are factors of safety for dead load and live load with values of 1.3 and 1.5 respectively.

2.2 Heterogeneous Bars Under Bending Stress (Bending Failure Mode)

The composite timber-aluminium column would behave as one in resisting bending induced in it as a result of bending moments. The two materials are rigidly connected as shown in Figure 1 and thus the strains in the two materials are same due to bending stresses at a section.

\[
P = f_T \left( A_T + \frac{A_A E_A}{E_T} \right)
\]

The aluminium gives a higher modulus of elasticity than the timber. For the composite timber-aluminium
column that is subjected to bending stresses, the
maximum stress in the composite section is given by
\[
\sigma_{\text{max}} = \frac{My_{\text{composite}}}{I_{\text{composite}}} \quad (10)
\]
The total stress in the composite column based on
individual stresses in each material
\[
\sigma_{\text{total}} = \sigma_T + \sigma_A \quad (11)
\]
Where; \(\sigma_T = \frac{My_T}{I_T}\) is stress in timber member and
\(\sigma_A = \frac{My_A}{I_A}\) is the stress in the aluminium component.

Since there is an interaction between the two
materials, the bending stresses will be distributed in
the ratio of the flexural rigidity of both materials. In
such case, the moment of inertia, \(I\), of the two
materials joined together is given by:
\[
I_{\text{composite}} = I_{\text{timber}} + I_{\text{aluminium}} \quad (12)
\]
Substituting into (10), the applied bending stress becomes
\[
\sigma = \frac{My_{\text{composite}}}{I_{\text{timber}} + I_{\text{aluminium}}} \quad (13)
\]
And since timber and aluminium have different
moduli of elasticity, the stresses in the compound
column will be distributed based on the modulus as
expressed below:
Bending stress in aluminium,
\[
\sigma = \frac{My_CE_A}{E_T I_T + E_A I_A} \quad (14)
\]
Bending stress in timber,
\[
\sigma = \frac{My_CE_T}{E_T I_T + E_A I_A} \quad (15)
\]
Hence, the total applied bending stress on the
composite column is given as:
\[
R = \frac{My_C}{E_T I_T + E_A I_A}(E_A + E_T) \quad (16)
\]
The applied moment on the composite column is
given by:
\[
M_{\text{applied}} = \frac{PL^2}{8} \quad (17)
\]
Where \(L\) is the column length and \(P\) is applied load
given as \(q_k(1.35\alpha + 1.5)\) where \(q_k\) is live load and \(\alpha\)
is dead to live load ratio, \(M\) is the moment acting on
the column due to lateral loads (beam-column), \(y\) is
the distance from the centroidal axis.
From the performance function \(G(x) = R - S\),
\[
G(x) = \left( \frac{My_C}{E_T I_T + E_A I_A}(E_A + E_T) \right) - 0.125Q_k(1.35\alpha + 1.5)L^2 \quad (18)
\]
2.3 Flexure Buckling Failure Mode
There is the tendency for the column to buckle in
bending due to axial load subjected on it. [10] gives
the flexural buckling of timber to satisfy the
interactive formula in the following equation,
\[
\left( \frac{\sigma_{c,d}}{k_{\text{crit},f,m,d}} \right)^2 + \left( \frac{\sigma_{c,0,d}}{k_{\text{crit},f,c,0,d}} \right)^2 \leq 1 \quad (19)
\]
From the performance function \(G(x) = R - S\),
\[
G(x) = 1 - \left( \frac{\sigma_{c,d}}{k_{\text{crit},f,m,d}} \right)^2 - \left( \frac{\sigma_{c,0,d}}{k_{\text{crit},f,c,0,d}} \right)^2 \quad (20)
\]
Where: \(\sigma_{c,d}\) is the bending stress, \(\sigma_{c,0,d}\) is the design
compressive stress parallel to grain, \(f_{m,d}\) is the
bending strength parallel to grain, \(f_{c,0,d}\) is the design
compressive strength parallel to grain.
\(k_{c,z}\) is the column instability factor given as:
\[
k_{c,z} = - \frac{1}{k_z + \sqrt{k^2_z - \lambda^2_{rel,z}}} \quad (21)
\]
Where: \(k_z = 0.5(1 + \beta_c(\lambda_{rel,z} - 0.3 + \lambda^2_{rel,z}))\) and \(\beta_c\)
is a factor for members within a define limit and is
0.2 for solid timber.

The relative slenderness ratio \(\lambda_{rel,z} = \frac{L_z}{\sqrt{\kappa_{c,z}}}\)
\(\lambda_z\) is the slenderness ratio in the z-axis

3. THE LIMIT STATE PRINCIPLE
The performance of an engineering structure
depends on the type and magnitude of the applied
load and the structural strength and stiffness [6]. It
is convenient to describe failure events in terms of
functional relations, which if they are fulfilled, define
that the failure event \(F\) will occur:
\[
f = (g(x) \leq 0) \quad (22)
\]
where \(g(x)\) is a limit state function, the components
of the vector \(x\) are basic random variables \(X\)
representing all relevant uncertainties influencing the
problem at hand. The failure event \(F\) is defined as
the set of realisations of the limit state function \(g(x)\),
which are zero or negative.
The First Order Reliability Method (FORM) is a level II
(reliability index method) analysis for solving
probability of failure where uncertain parameters are
modelled by the mean values and the standard
deviations, and by the correlation coefficients
between stochastic variables [5, 11]. FORM involves
the use of stochastic variables and models, where the
stochastic variables are denoted \(X = (X_1, \ldots, X_n)\).
The \(n\) stochastic variables could model physical
uncertainty, model uncertainty or statistical
uncertainties.
The application of FORM gives the state of the
structure; whether the structure is in a safe state or
in a failure state. The basic variable space is divided,
by the failure state (limit state surface), into two sets: the safe and the failure set.
The failure surface is expressed by the equation:

\[ g(x) = g(x_1, \ldots, x_n) = 0 \] (23)

Where \( g(x) \) is the failure function
If \( R \) is the resistance and \( S \) the effect of actions, the performance function \( g \) is given as [12][13]:

\[ g = R - S \] (24)

\( R, S \) and \( g \) are random variables.
If the performance function, \( g \), is normally distributed, the expected value and standard deviation can be expressed as:

\[ \beta = \frac{\mu_g}{\sigma_g} \] (25)

\( \beta \) is the Hasofer & Lind reliability index and it is defined as the smallest distance from the origin \( O \) in the \( u \)-space to the failure surface \( g(x) = 0 \).
And

\[ P_f = P(g(x) \leq 0) = P(\mu_g - \beta \sigma_g \leq 0) = P\left( U \leq -\frac{\mu_g}{\sigma_g} \right) \]

\( \Phi \) is the standard normal distribution function and \( U \) is a standard normally distributed variable with expected value zero and unit standard deviation \( (\mu_g = 0, \sigma_g = 1) \).

For a linear failure function, \( M \), if the stochastic variables \( P \) and \( S \) are independent, then the index becomes:

\[ \beta = \frac{\mu_R - \mu_S}{\sigma_R - \sigma_S} \] (27)

For other distributions of \( g \), \( \beta \) is only a conventional measure of the reliability \( P_s = (1 - P_f) \).

4. METHODS
The assumed cross sections replicate usual conditions and dead loads are established according to [14]. The modification factor takes into account the duration of load effect and moisture content and its value is considered as constant and equal to \( k_{mod} = 0.60 \), as described in [10]. The reliability-based analyses were carried out with a computer program written in MATLAB. The computer program performs the reliability analysis of axially loaded solid slender columns. Three failure modes were considered as follows: compression criterion (failure mode I), bending criterion (failure mode II) and flexural-buckling (failure mode III).

4.1 Limit State Structural Design Parameters
The column used in the reliability analysis is considered as an axially loaded solid rectangular column with varying aluminium laminates at the weaker axis of buckling.
i. For the load model, the factors of safety are given as: \( k_s = 1.5 \) for imposed load and \( k_s = 1.35 \) for dead load

ii. The timber column is considered a structural member in a dwelling and it is considered as permanent structure. Hence, the strength modification, \( k_{mod} \) is 0.6 [10].

iii. The properties of timber that were obtained from [15] were used for the basic variables whereas the statistical parameters and distribution models were obtained from [16].

4.2 Stochastic Models for the Basic Variables
The values of data used in the models are presented in Tables 1, 2 and 3.

<table>
<thead>
<tr>
<th>S/no</th>
<th>Variables</th>
<th>Meaning</th>
<th>Distribution</th>
<th>Mean</th>
<th>Covariance</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f_{c,0}(\text{itako}) )</td>
<td>Characteristic compressive strength</td>
<td>lognormal</td>
<td>29.58 N/mm²</td>
<td>0.13</td>
<td>3.84</td>
</tr>
<tr>
<td>2</td>
<td>( f_{c,0}(\text{Oporoporo}) )</td>
<td>compressive strength parallel to grain</td>
<td>lognormal</td>
<td>20.82 N/mm²</td>
<td>0.13</td>
<td>2.71</td>
</tr>
<tr>
<td>3</td>
<td>( f_{c,0}(\text{opepe}) )</td>
<td>compressive strength parallel to grain</td>
<td>lognormal</td>
<td>27.18 N/mm²</td>
<td>0.13</td>
<td>3.53</td>
</tr>
<tr>
<td>4</td>
<td>( f_{c,0}(\text{ijebu}) )</td>
<td>modulus of elasticity</td>
<td>lognormal</td>
<td>24.16 N/mm²</td>
<td>0.13</td>
<td>3.14</td>
</tr>
<tr>
<td>5</td>
<td>( f_{c,0}(\text{aluminium}) )</td>
<td>modulus of elasticity</td>
<td>lognormal</td>
<td>100 N/mm²</td>
<td>0.13</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Qk</td>
<td>Imposed load</td>
<td>Gumbel</td>
<td>15kN</td>
<td>0.32</td>
<td>4.8</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>Width of column</td>
<td>Normal</td>
<td>300mm</td>
<td>0.47</td>
<td>141</td>
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<tr>
<td>8</td>
<td>H</td>
<td>Thickness of column</td>
<td>Normal</td>
<td>100mm</td>
<td>0.47</td>
<td>47</td>
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<td>9</td>
<td>( E_{\text{itako}} )</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>9.18 kN/mm²</td>
<td>0.21</td>
<td>1.93</td>
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<td>10</td>
<td>( E_{\text{Oporoporo}} )</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>4.95 kN/mm²</td>
<td>0.21</td>
<td>1.04</td>
</tr>
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<td>11</td>
<td>( E_{\text{opepe}} )</td>
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<td>Lognormal</td>
<td>8.42 kN/mm²</td>
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<td>( E_{\text{ijebu}} )</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>7.36 kN/mm²</td>
<td>0.21</td>
<td>1.55</td>
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<td>13</td>
<td>( E_{\text{aluminium}} )</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>70 kN/mm²</td>
<td>0.21</td>
<td>14.7</td>
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</tbody>
</table>
Table 2: Stochastic Parameters for Bending Failure Mode

<table>
<thead>
<tr>
<th>S/no</th>
<th>Variables</th>
<th>Meaning</th>
<th>Distribution</th>
<th>Mean</th>
<th>Covariance</th>
<th>SD</th>
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<tbody>
<tr>
<td>1</td>
<td>$f_{c,90,k}$ (itako)</td>
<td>Characteristic bending strength</td>
<td>Lognormal</td>
<td>51.97 N/mm²</td>
<td>0.13</td>
<td>6.76</td>
</tr>
<tr>
<td>2</td>
<td>$f_{c,90,k}$ (Oporoporo)</td>
<td>Characteristic bending strength</td>
<td>Lognormal</td>
<td>32.30 N/mm²</td>
<td>0.13</td>
<td>4.2</td>
</tr>
<tr>
<td>3</td>
<td>$f_{c,90,k}$ (opepe)</td>
<td>bending strength perpendicul to grain</td>
<td>Lognormal</td>
<td>23.80 N/mm²</td>
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<td>3.1</td>
</tr>
<tr>
<td>4</td>
<td>$f_{c,90,k}$ (jebu)</td>
<td>bend strength perpendicul to grain</td>
<td>Lognormal</td>
<td>33.12 N/mm²</td>
<td>0.13</td>
<td>4.3</td>
</tr>
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<td>5</td>
<td>$f_{c,90,k}$ (aluminium)</td>
<td>bend strength perpendicul to grain</td>
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<td>100 N/mm²</td>
<td>0.13</td>
<td>13</td>
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<td>Qk</td>
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<td>47</td>
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<td>9</td>
<td>L</td>
<td>Column height</td>
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<td>$E_{itako}$</td>
<td>Modulus of elasticity</td>
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<td>$E_{Oporoporo}$</td>
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<td>$E_{aluminium}$</td>
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<td>70 kN/mm²</td>
<td>0.21</td>
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Table 3: Stochastic Parameters for Flexural Failure Mode

<table>
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<tr>
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<td>8</td>
<td>$f_{c,90,k}$ (opepe)</td>
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<td>23.80 N/mm²</td>
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<td>3.1</td>
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<td>$f_{c,90,k}$ (jebu)</td>
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<td>10</td>
<td>$f_{c,90,k}$ (aluminium)</td>
<td>Characteristic compressive strength</td>
<td>Lognormal</td>
<td>100 N/mm²</td>
<td>0.13</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>Qk</td>
<td>Imposed load</td>
<td>Gumbel</td>
<td>15kN</td>
<td>0.32</td>
<td>4.8</td>
</tr>
<tr>
<td>12</td>
<td>B</td>
<td>Width of column</td>
<td>Normal</td>
<td>300mm</td>
<td>0.47</td>
<td>141</td>
</tr>
<tr>
<td>13</td>
<td>H</td>
<td>Thickness of column</td>
<td>Normal</td>
<td>100mm</td>
<td>0.47</td>
<td>47</td>
</tr>
<tr>
<td>14</td>
<td>L</td>
<td>Column height</td>
<td>Normal</td>
<td>3500mm</td>
<td>0.28</td>
<td>980</td>
</tr>
<tr>
<td>15</td>
<td>$E_{itako}$</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>10.0 kN/mm²</td>
<td>0.21</td>
<td>1.93</td>
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<td>16</td>
<td>$E_{Oporoporo}$</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>6.67 kN/mm²</td>
<td>0.21</td>
<td>1.04</td>
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<td>9.28 kN/mm²</td>
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<td>1.77</td>
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<tr>
<td>18</td>
<td>$E_{jebu}$</td>
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<td>8.50 kN/mm²</td>
<td>0.21</td>
<td>1.55</td>
</tr>
<tr>
<td>19</td>
<td>$E_{aluminium}$</td>
<td>Modulus of elasticity</td>
<td>Lognormal</td>
<td>70 kN/mm²</td>
<td>0.21</td>
<td>14.7</td>
</tr>
</tbody>
</table>

5. DISCUSSION OF RESULTS
The results for the reliability analysis are presented in Figures 3 to 12 and all results show a general improvement in the safety (reliability) indices of the column with increase in laminate thickness but a decline in safety indices with increase in load ratios. The performance of the structure shows varied safety indices for the three modes of failure and for the design parameters that were inputted into the limit state functions.

Figure 3: Safety Index Versus Laminate Thickness at Various imposed Loads for Strombosis Pustulata
5.1.1 Results for Compression Mode of Failure

The results for the compression mode of failure show that the column is very safe in compression with all varied parameters having safety indices well above the target safety index of 3.8. Figure 3 shows the effects of varying the imposed loads on the safety index of the column. Figure 4 shows the effect of varying dead-to-live load ratios on the safety index of the column with lowest safety indices of 14.96, highest $\beta$ of 22.77 without laminate and minimum safety indices of 27.26 and highest $\beta$ of 33.73 for the timber column with 20mm thick aluminium laminate. It can be observed that the disparity between the safety indices for load ratios is not as large as that of varying the imposed load. Figure 5 shows the effects of the inherent strength of the timber species on the safety indices of the column using the all the standard design parameters. It will be observed that varying the imposed loads has a greater effect on the safety index with a change in $\beta$ from 4.3 to 17.4 for the timber column without laminates and from 12.4 to 29.6 for the column with laminate thickness of 20mm. The effect of the timber compressive strength is also evident in Figure 5 with the big disparity in the safety indices for the four timber species. It can be observed that itako timber specie gave a large safety index of 11.4 without laminate and 23.2 with laminate of 20mm thickness.

5.1.2 Results for Bending Mode of Failure

The results of the safety indices for the bending criterion of failure for the laminated timber column is shown in Figures 6 to 9 with the varied parameters being imposed load, load ratios, column height and timber bending strength.

Figure 6 shows the effects of load variation on the safety of timber columns in bending. It can be observed that the timber column will fail for all imposed loads except for 10kN applied load where the safety index is 0.32 for column without laminate and 3.38 for laminate with thickness of 20mm. Also, the laminate thickness of 10mm, gives a safe column for imposed load of 15kN and 20kN. Figure 7 gives the effect of varying dead-to-live load ratio on the safety of the column under bending loads. It can be observed that the column is in safe zone for all load ratios and exceeds the target safety index for load ratios of 0.2 and 0.4 for laminate with thickness of 12mm and 16mm respectively. Figure 8 shows the effect the height of column will have on the safety of the column. The column with height of 1m is very safe with safety index of 3.8 without laminate, from where it increases to a safety index of 9.01 for a laminate of 20mm thickness. Most of the column heights are safe except for 3m and 3.5m where it is
safe only from the point of application of laminate of 5mm thickness.
It is also observed in Figure 8 that the change in length gave the most critical state in the change of the safety indices of the column while variations in load ratios have the least impact on the column safety index. As such the design of the column should be more centred on the column height than other parameters in bending failure mode.

5.1.3 Results for flexural buckling mode of failure
Figures 10 to 12 show the safety indices of the timber column for the flexural buckling mode of failure of columns. The charts show that varying loads and column height are the critical parameters that affect the safety indices of the column in this mode of failure.
Figure 10 shows the result of varying imposed loads on the safety of the laminate timber columns. It will be observed that the column fails for imposed loads of 20kN, 25kN and 30kN at all laminate thickness. The column is safe for 15kN when the laminate of 12mm thickness is used and is safe for all values of 10kN with the highest safety index being 2.14 for laminate of 20mm thickness. Figure 11 shows the result of dead-to-live load ratios on the safety of the composite column where the column is safe when a laminate thickness of 10mm is used for the column in the case of load ratio of 1.0 and 4mm for load ratios of 0.6 and 0.8 while all other load ratios gave a safe design. Figure 12 shows the safety index based on the height of column for flexural buckling. The result shows that the column is safe for heights of 1m and 1.5m, but for greater heights, the laminate thickness of 12mm is used to ensure safety. Also, the column height of 1m should be used with 8mm thick laminate and 16mm thick laminates for 1.5mm column height to ensure it meets up with the target reliability index of 3.8 as stipulated in [11] and [12].

6. CONCLUSION
Reliability analyses of a laminated timber column with varying aluminium laminate thickness were carried out. The effects of varying loads, timber strengths (compressive and bending) and column height on the safety indices were considered with varying thickness of aluminium laminate. The results of the study show that for a timber column, the effect of change in imposed loads had a great effect on the safety indices of columns for both compression and bending but more critical in bending. Also the effect of load is more profound on the safety index of columns in bending while the effect of change in load ratio showed little effect on the overall change of safety index. It is observed in the charts that the aluminium laminate greatly increased the strength of the column and hence gave a favourable increase in the safety indices for all the failure criteria.

7. REFERENCES