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PREDICTING THE SPLIT TENSILE STRENGTH OF SAW DUST ASH - FINE AGGREGATE CONCRETE

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ABSTRACT
The industrial waste, Saw Dust Ash (SDA), has played a key role in concrete mix research. It has served as an alternative or complementary material to some of the traditional materials of concrete. In this study, SDA was used to replace 5% of the fine aggregate (sand), as the other three ingredients, cement, granite, and water remained constant. Scheffe's simplex lattice was used for five mix ratios in a {5,2} component mix, which resulted in additional ten mix ratios. Additional fifteen mix ratios were generated for verification and testing. The thirty concrete mix ratios were subjected to laboratory experiments to determine the 28 days Split Tensile Strengths. The results of the first fifteen Split Tensile strengths were used for the calibration of the model constant coefficients using Scheffe's simplex approach, while those from the second fifteen were used for the model verification. A mathematical regression model was derived from the experimental results, with which the Split Tensile Strengths were predicted. The derived model was subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate with an R^2 value of 0.8099. The study revealed that SDA can replace 5% of fine aggregate and promote sustainability, without compromising the 28 days Split Tensile Strength.

Keywords: *Saw Dust Ash, Scheffe's simplex lattice, Split Tensile Strength of concrete.*

1. INTRODUCTION

Industrial waste materials in concrete research and construction has been in use for quite some time. Such materials as fly ash, saw dust ash (SDA), rice husk ash, quarry dust, and palm kernel shell ash are some of the industrial wastes often used in recent times. They have been used in one form or another, to replace fractions of either cement or fine aggregates, while others have been used to stabilise sub-base materials for pavement construction [4].
Saw dust is an industrial waste or bye-product of saw mills produced after the wood has been sawn to shape in a saw mill, and comes out in powder form. It has been used in concrete construction for over 30 years [1]. When saw dust is subjected to fire, it burns to ashes. That ash is called Saw Dust Ash (SDA).

In this study, SDA was used to partially replace 5% of the fine aggregate. A mathematical model was derived using Scheffe's simplex theory, with which the Split tensile strengths were predicted. There were five components in the mix (water-cement ratio, cement, sand, SDA, and granite).

2. LITERATURE REVIEW

The biggest material role in the construction industry, according to [2] is played by concrete. Several authors have studied and determined various means of actualizing sustainability in the construction industry with respect to concrete, using SDA. Ogunribido [3] demonstrated in his experimental research that SDA can drastically improve the properties of lateritic soils when used as a stabiliser. From his findings, the optimum moisture content,

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maximum dry density, compressive strength, and shear strength of the lateritic soil were improved when stabilised with SDA. A similar study carried out by [4] also shows that using saw dust to stabilise lateritic soils could improve the CBR and other properties of the soil, as well as reduce the construction cost. According to [5], cement is a major source of environmental degradation, as about 400kg of CO_2 is being emitted for every 600kg of cement produced, therefore replacing 10% of cement with SDA does not negatively affect the chloride permeability and thaw resistance of the concrete, but decreases the drying shrinkage, and increases the water absorption. This also established the pozzolanic ability of SDA. Similarly, [6] found in their research that replacing 5 to 15% cement content with saw dust increased the compressive, flexural, and split tensile strengths of the concrete for 28days curing period and beyond. It also decreased the weight and cost. However, fewer researchers such as, [7] have carried out research work on replacement of fine aggregates with SDA. Their research findings revealed that 10% replacement of fine aggregate with SDA will result in an acceptable tensile, flexural, and compressive strengths as well as reduce the amount of wastes in the environment.

SDA has different particles that are mostly angular in shape [7]. According to [7] SDA has a specific gravity of 2.5, fineness modulus of 1.78, water absorption of 0.56%, and bulk dry density of 1300kg/m^3 as against sand with specific gravity of 2.65, fineness modulus of 2.21, water absorption of 0.45%, and bulk dry density of 1512 kg/m^3. When 10% of SDA was added to the sand, these properties became 2.67, 2.2, 0.5%, and 1436kg/m^3 for specific gravity, fineness modulus, water absorption, and bulk dry density respectively. This significantly indicates that the mixture of sand and 10% SDA replacement gave similar physical properties with the 0% SDA replacement, making the mixture adequate for a fine aggregate. However, from the study of [8] SDA had a specific gravity of 2.19, bulk dry density of 1040kg/m^3, and moisture content of 0.3%; which gave a bigger difference in the specific gravity of SDA as compared to that of sand. Furthermore, [9] showed that 50% of the SDA grain size passed through the AASHTO sieve no. 200 (75μm) while 31% was retained by sieve no. 325 (45 μm); which justifies the fineness of SDA.

SDA, like many other concrete construction materials, contains several chemical compounds. According to [7] SDA has the following chemical composition by mass: 65.3% SiO_2, 4% Al_2O_3, 2.23% Fe_2O_3, 9.6% CaO, 5.8% MgO, 0.01% MnO, 0.07% Na_2O, 0.11% K_2O, 0.43% P_2O_5, and 0.45% SO_2. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 71.53%. Similar works carried out by [10] reveals 67.95% SiO_2, 4.29% Al_2O_3, 2.15% Fe_2O_3, 9.47% CaO, 5.84% MgO, 0.01% MnO, 0.06% Na_2O, 0.11% K_2O, and 0.56% SO_3. Summing up SiO_2, Al_2O_3, and Fe_2O_3 gives 74.39. These, in accordance with [11] indicate that SDA is a good pozzolanic material. The chemical compositions as found by [7]; [8]; [10] all show that SDA has a high percentage of SiO_2 and small percentages of Al_2O_3 and Fe_2O_3, which are similar to those of sand with high percentage of about 95% SiO_2. Hence SDA can be used with sand as fine aggregate.

2.1. Concrete Mix Design

There are several methods of concrete mix design. In the United States the American Concrete Institute (ACI) method and the United States Bureau of Reclamation (USBR) method are used. In the United Kingdom and many parts of the world, the Building Research Establishment (BRE) method is used while the Bureau of Indian Standards (BIS) method is used in India.

2.2. Scheffe's Simplex Theory

Several authors such as [12]; [13]; [14]; [15]; [16]; [17]; and [18] have carried out concrete mixture research with the development of mathematical models. Most of such works were based on Scheffe's Simplex theory.

Scheffe's model is based on the simplex lattice and simplex theory or approach [19]. The simplex approach considers a number of components, *q*, and a degree of polynomial, *m*. The sum of all the i^{th} components is not greater than 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \tag{1}$$

$$\boldsymbol{x_1 + x_2 + \cdots + x_q = 1} \tag{2}$$

with $0 \leq x \leq 1$. The factor space becomes S_{q-1}. According to [19] the $\{q,m\}$ simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of $x_i = 0, {}^{1}/_{m}, {}^{2}/_{m}, \ldots {}^{m}/_{m}$. For a 3-

component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is x_i = 0, $^1/_2$, and 1 and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a {*5,2*} simplex, there will be 15 points with x_i = 0, $^1/_2$, and 1 as spaced values. The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,0,1/2,1/2,0), (0,0,0,1/2,1/2), (0,1/2,0,1/2,0), (0,0,1/2,0,1/2), (0,1/2,0,0,1/2).

$$N = C_n^{(q+n-1)} \quad (3)$$

$$or\ N = \frac{(q+n-1)!}{(q-1)!\,(n)!} \quad (4)$$

For a polynomial of degree *m* with *q* component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_i\, x_i + \sum b_{ij}\, x_i x_j + \sum b_{ijk}\, x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in}\, x_{i1} x_{i2} x_{in} \quad (5)$$

Where 1 ≤ *i* ≤ *q*, 1 ≤ *i* ≤ *j* ≤ *q*, 1 ≤ *i* ≤ *j* ≤ *k* ≤ *q*, and b_0 is the constant coefficient.

x is the pseudo component for constituents *i*, *j*, and *k*.

When {*q,m*} = {*5,2*}, eq. (5) becomes:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 \quad (6)$$

and eq. (2) becomes

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (7)$$

Multiplying eq. (7) by b_0 gives

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \quad (8)$$

Multiplying eq. (7) successively by x_1, x_2, x_3, x_4, and x_5 and making x_1, x_2, x_3, x_4, and x_5 the subjects of the respective formulas:

$$\begin{aligned} x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5 \\ x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5 \\ x_3^2 &= x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \\ x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5 \\ x_5^2 &= x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5 \end{aligned} \quad (9)$$

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$Y = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5) + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5) + b_{55}(x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5)$$

$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_0 + b_5 + b_{55})x_5 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{15} - b_{11} - b_{55})x_1x_5 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{25} - b_{22} - b_{55})x_2x_5 + (b_{34} - b_{33} - b_{44})x_3x_4 + (b_{35} - b_{33} - b_{55})x_3x_5 + (b_{45} - b_{44} - b_{55})x_4x_5 \quad (10)$$

Let

$$\begin{aligned} &\beta_1 = b_0 + b_1 + b_{11};\ \beta_2 = b_0 + b_2 + b_{22}; \\ &\beta_3 = b_0 + b_3 + b_{33};\ \beta_4 = b_0 + b_4 + b_{44} \\ &\beta_5 = b_0 + b_5 + b_{55};\ \beta_{12} = b_{12} - b_{11} - b_{22} \\ &\beta_{13} = b_{13} - b_{11} - b_{33};\ \beta_{14} = b_{14} - b_{11} - b_{44} \\ &\beta_{15} = b_{15} - b_{11} - b_{55};\ \beta_{23} = b_{23} - b_{22} - b_{33} \\ &\beta_{24} = b_{24} - b_{22} - b_{44};\ \beta_{25} = b_{25} - b_{22} - b_{55} \\ &\beta_{34} = b_{34} - b_{33} - b_{44};\ \beta_{35} = b_{35} - b_{33} - b_{55} \\ &\beta_{45} = b_{45} - b_{44} - b_{55} \end{aligned} \quad (11)$$

Substituting eq. (11) into eq. (10) gives

$$Y = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{15}x_1x_5 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{25}x_2x_5 + \beta_{34}x_3x_4 + \beta_{35}x_3x_5 + \beta_{45}x_4x_5 \quad (12)$$

This can be rewritten as:

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \le i \le j \le 5} \beta_{ij} x_i x_j \quad (13)$$

Where the response, Y is a dependent variable (tensile strength of concrete). Hence, eq. (12) is the general equation for a {*5,2*} polynomial, and it has 15 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in *i* and *j*. If x_i=1 and x_j = 0, since $j \neq i$, then

$$\boldsymbol{Y_i = \beta_i} \quad (14)$$

Which means

$$\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i \qquad (15)$$

Hence, from eq. (14)

$$Y_1 = \beta_1 ;\ Y_2 = \beta_2 \ ;\ Y_3 = \beta_3 \ ;\ Y_4 = \beta_4 \ ;\ Y_5 = \beta_5 \qquad (16)$$

According to [19],

$$\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j \qquad (17)$$

Substituting eq. (14)

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \qquad (18)$$

3. MATERIALS AND METHODS

Water, cement, sand, SDA, and granite were the materials used to produce the concrete.

The first five concrete mix ratios derived from different mix design methods given as

BRE 12 = [0.54 1 1.9475 0.1025 2.95];

BRE 22 = [0.58 1 2.1185 0.1115 3.21];

USBR 22 = [0.58 1 2.2515 0.1185 3.29];

BIS 12 = [0.43 1 1.2065 0.0635 2.88];

ACI 12 = [0.55 1 1.8335 0.0965 3.09]

These can be transposed and put in matrix form as follows:

$$S = \begin{bmatrix} 0.54 & 0.58 & 0.58 & 0.43 & 0.55 \\ 1 & 1 & 1 & 1 & 1 \\ 1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335 \\ 0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965 \\ 2.95 & 3.21 & 3.29 & 2.88 & 3.09 \end{bmatrix} (19)$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (20)$$

With centre points

X_{12} = [0.5 0.5 0 0 0]; X_{13} = [0.5 0 0.5 0 0];

X_{14} = [0.5 0 0 0.5 0]; X_{15} = [0.5 0 0 0 0.5];

X_{23} = [0 0.5 0.5 0 0]; X_{24} = [0 0.5 0 0.5 0];

X_{25} = [0 0.5 0 0 0.5]; X_{34} = [0 0 0.5 0.5 0];

X_{35} = [0 0 0.5 0 0.5]; X_{45} = [0 0 0 0.5 0.5]

According to Scheffe (1958),

$$S_{ij} = XS_i \qquad (21)$$

Substituting,

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{14} \\ S_{15} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.54 \\ 0.58 \\ 0.58 \\ 0.43 \\ 0.55 \end{bmatrix} \qquad (22)$$

This process is repeated for S_{24}, S_{25}, S_{34}, S_{35}, and S_{45}. Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular pentagons for the actual components with their corresponding pseudo components are given in figures (1) and (2) respectively.

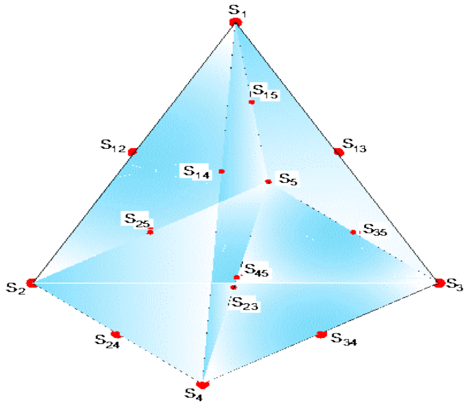

Fig. 1: Simplex Plot for Actual Comp

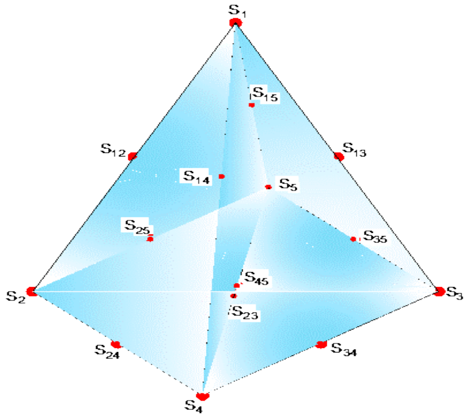

Fig. 2: Simplex Plot for Pseudo Components

3.1. Split Tensile Test

The concrete samples were prepared in cylindrical shapes of 300mm X 150mm diameter. The split tensile test which is the most commonly used indirect tensile test was used to determine the tensile strength of the concrete. The specimens were subjected to a compressive load along the vertical diameter at a constant rate. This brought about a tensile split in the specimen. The tensile strength is then determined by,

$$f_t = \frac{2P}{\pi l d} \qquad (23)$$

Where P = the load at failure (KN)

d = the diameter of the specimen in millimetres

l = the span length of specimen in millimetres

Two replicates were made, and the average taken and recorded (see table 4).

Table 1: Model Mix Ratios

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
BRE12	0.54	1	1.9475	0.1025	2.95	Y_1	1	0	0	0	0
BRE22	0.58	1	2.1185	0.1115	3.21	Y_2	0	1	0	0	0
USBR22	0.58	1	2.2515	0.1185	3.29	Y_3	0	0	1	0	0
BIS12	0.43	1	1.2065	0.0635	2.88	Y_4	0	0	0	1	0
ACI12	0.55	1	1.8335	0.0965	3.09	Y_5	0	0	0	0	1
N1	0.56	1	2.033	0.107	3.08	Y_{12}	0.5	0.5	0	0	0
N2	0.56	1	2.0995	0.1105	3.12	Y_{13}	0.5	0	0.5	0	0
N3	0.485	1	1.577	0.083	2.915	Y_{14}	0.5	0	0	0.5	0
N4	0.545	1	1.8905	0.0995	3.02	Y_{15}	0.5	0	0	0	0.5
N5	0.58	1	2.185	0.115	3.25	Y_{23}	0	0.5	0.5	0	0
N6	0.505	1	1.6625	0.0875	3.045	Y_{24}	0	0.5	0	0.5	0
N7	0.565	1	1.976	0.104	3.15	Y_{25}	0	0.5	0	0	0.5
N8	0.505	1	1.729	0.091	3.085	Y_{34}	0	0	0.5	0.5	0
N9	0.565	1	2.0425	0.1075	3.19	Y_{35}	0	0	0.5	0	0.5
N10	0.49	1	1.52	0.08	2.985	Y_{45}	0	0	0	0.5	0.5

Table 2: Control Points

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
C1	0.558	1	2.0463	0.1077	3.114	Y_{C1}	0.4	0	0.4	0	0.2
C2	0.52	1	1.7537	0.0923	3.078	Y_{C2}	0	0.6	0	0.4	0
C3	0.548	1	2.0083	0.1057	3.018	Y_{C3}	0.8	0	0.2	0	0
C4	0.49	1	1.5713	0.0827	3.012	Y_{C4}	0	0.4	0	0.6	0
C5	0.544	1	1.9019	0.1001	3.006	Y_{C5}	0.6	0	0	0	0.4
C6	0.55	1	2.0425	0.1075	3.208	Y_{C6}	0	0	0.8	0.2	0
C7	0.55	1	1.9589	0.1031	3.03	Y_{C7}	0.6	0.2	0	0	0.2
C8	0.514	1	1.6967	0.0893	3.054	Y_{C8}	0	0.4	0	0.4	0.2
C9	0.548	1	1.8563	0.0977	3.062	Y_{C9}	0.2	0	0	0	0.8
C10	0.46	1	1.4155	0.0745	2.962	Y_{C10}	0	0	0.2	0.8	0
C11	0.566	1	2.1071	0.1109	3.182	Y_{C11}	0.2	0	0.6	0	0.2
C12	0.544	1	1.9323	0.1017	3.152	Y_{C12}	0	0.2	0.4	0.2	0.2
C13	0.58	1	2.1451	0.1129	3.226	Y_{C13}	0	0.8	0.2	0	0
C14	0.532	1	1.7651	0.0929	3.072	Y_{C14}	0	0.2	0	0.2	0.6
C15	0.536	1	1.8715	0.0985	3.084	Y_{C15}	0.2	0.2	0.2	0.2	0.2

4. RESULTS AND DISCUSSIONS

4.1. Scheffe's Model for 28 days Split Tensile Strength

The coefficients of polynomial with the aid of table (4), eq. (16), and eq. (18) are:

$\beta_1 = 2.987$, $\beta_2 = 2.378$, $\beta_3 = 2.625$, $\beta_4 = 3.799$, $\beta_5 = 3.176$, $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$

$\beta_{12} = 4 \times 3.069 - 2 \times 2.987 - 2 \times 2.625 = 1.546$

Similarly, $\beta_{13} = -2.336$, $\beta_{14} = -2.836$, $\beta_{15} = -0.086$, $\beta_{23} = 1.418$, $\beta_{24} = -0.666$, $\beta_{25} = 0.36$, $\beta_{34} = -0.704$, $\beta_{35} = -2.19$, $\beta_{45} = -2.714$.

Substituting the above coefficients into eq. (12) gives

$$Y = 2.987x_1 + 2.378x_2 + 2.625x_3 + 3.799x_4 + 3.176x_5 + 1.546x_1x_2 - 2.336x_1x_3 - 2.836x_1x_4 - 0.086x_1x_5 + 1.418x_2x_3 - 0.666x_2x_4 - 0.36x_2x_5 - 0.704x_3x_4 - 2.19x_3x_5 - 2.714x_4x_5 \quad (24)$$

Eq. (24) above is the mathematical model to predict the 28 days split tensile strength of concrete using SDA to replace 5% of fine aggregate.

Table 3: Sieve Analysis Data for Fine Aggregate with 5% SDA replacement

Standard Sieve Opening Sizes			Mass Retained (g)	Cumulative Mass Retained (g)	% Retained	Cumulative % Retained	% Passing
Sieve Number	Sieve size (in)	Sieve size (mm)					
1/4"	0.25	6.3	0	0	0.00%	0.00%	100.00%
#4	0.187	4.75	2.2	2.2	0.73%	0.73%	99.27%
#8	0.0929	2.36	10.6	12.8	3.51%	4.24%	95.76%
#16	0.0465	1.18	29.7	42.5	9.84%	14.08%	85.92%
#30	0.0236	0.6	89.1	131.6	29.52%	43.61%	56.39%
#50	0.0118	0.3	111.3	242.9	36.88%	80.48%	19.52%
#100	0.00591	0.15	52.5	295.4	17.40%	97.88%	2.12%
#200	0.00295	0.075	6.1	301.5	2.02%	99.90%	0.10%
Pan	0	0	0.3	301.8	0.10%	100.00%	
		Total mass	301.8				

Fineness modulus, $FM = \frac{0.73+4.24+14.08+43.61+80.48+97.88}{100} = 2.41$

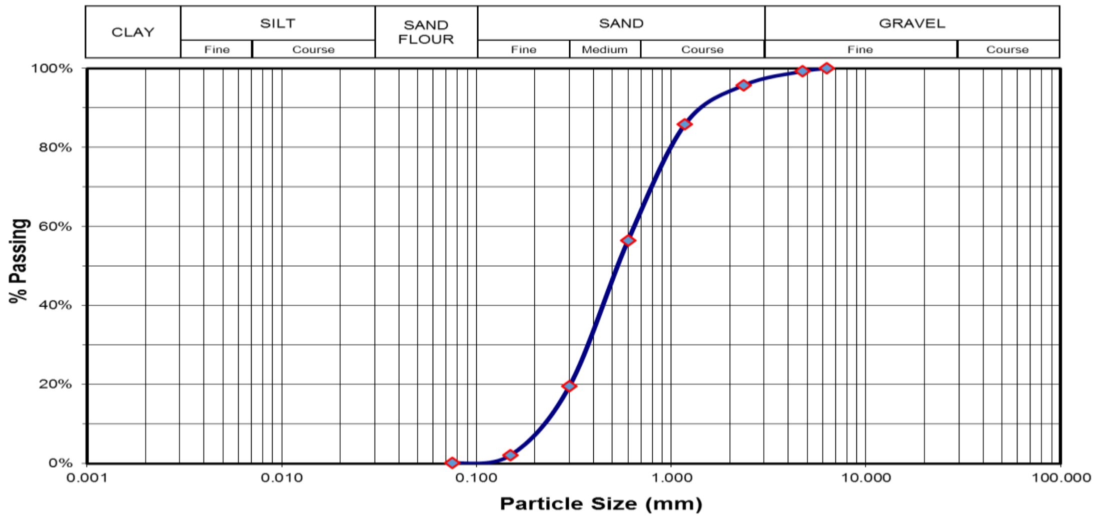

Fig. 3: Particle Size Distribution for Fine Aggregate with 5% SDA replacement

Table 4: Split Tensile Strength of Concrete

Sample	Curing	Load (KN)		$\frac{2}{\pi L d}$	Split tensile Strength (N/mm^2)		
		A	B		A	B	Average
BRE12	7 Days	145.96	179.82	14.1471	2.065	2.544	2.304
	28 Days	221.17	201.06	14.1471	3.129	2.844	2.987
BRE22	7 Days	165.88	165.73	14.1471	2.347	2.345	2.346
	28 Days	163.36	172.79	14.1471	2.311	2.444	2.378
USBR22	7 Days	121.58	152.34	14.1471	1.720	2.155	1.938
	28 Days	185.13	186	14.1471	2.619	2.631	2.625
BIS12	7 Days	203.43	203.43	14.1471	2.878	2.878	2.878
	28 Days	284.2	252.9	14.1471	4.021	3.578	3.799
ACI12	7 Days	191.97	195.87	14.1471	2.716	2.771	2.743
	28 Days	223.12	225.88	14.1471	3.157	3.196	3.176
N1	7 Days	188.81	169.65	14.1471	2.671	2.400	2.536
	28 Days	207.35	226.55	14.1471	2.933	3.205	3.069
N2	7 Days	150.76	138.23	14.1471	2.133	1.956	2.044
	28 Days	155.19	158.96	14.1471	2.195	2.249	2.222
N3	7 Days	141.37	186.23	14.1471	2.000	2.635	2.317
	28 Days	189.06	190.38	14.1471	2.675	2.693	2.684
N4	7 Days	154.57	156.45	14.1471	2.187	2.213	2.200
	28 Days	222.42	210.17	14.1471	3.147	2.973	3.060

Sample	Curing	Load (KN)		$\frac{2}{\pi Ld}$	Split tensile Strength (N/mm^2)		
		A	B		A	B	Average
N5	7 Days	165.88	182.65	14.1471	2.347	2.584	2.465
	28 Days	207.66	196.04	14.1471	2.938	2.773	2.856
N6	7 Days	132.26	165.25	14.1471	1.871	2.338	2.104
	28 Days	204.83	208.29	14.1471	2.898	2.947	2.922
N7	7 Days	144.2	173.1	14.1471	2.040	2.449	2.244
	28 Days	199.84	205.46	14.1471	2.827	2.907	2.867
N8	7 Days	169.33	148.27	14.1471	2.396	2.098	2.247
	28 Days	219.28	209.86	14.1471	3.102	2.969	3.036
N9	7 Days	125.05	182.38	14.1471	1.769	2.580	2.175
	28 Days	158.56	174.04	14.1471	2.243	2.462	2.353
N10	7 Days	138.86	172.11	14.1471	1.964	2.435	2.200
	28 Days	199.49	197.61	14.1471	2.822	2.796	2.809
C1	7 Days	139.8	157.93	14.1471	1.978	2.234	2.106
	28 Days	170.3	172.1	14.1471	2.409	2.435	2.422
C2	7 Days	139.96	145.33	14.1471	1.980	2.056	2.018
	28 Days	189.66	189.5	14.1471	2.683	2.681	2.682
C3	7 Days	151.45	155.19	14.1471	2.143	2.195	2.169
	28 Days	182	183.31	14.1471	2.575	2.593	2.584
C4	7 Days	140.75	152.46	14.1471	1.991	2.157	2.074
	28 Days	219.91	226.51	14.1471	3.111	3.204	3.158
C5	7 Days	157.39	157.08	14.1471	2.227	2.222	2.224
	28 Days	204.2	200.75	14.1471	2.889	2.840	2.864
C6	7 Days	140.64	148.04	14.1471	1.990	2.094	2.042
	28 Days	201.47	201	14.1471	2.850	2.844	2.847
C7	7 Days	153.94	166.22	14.1471	2.178	2.352	2.265
	28 Days	197.92	202.63	14.1471	2.800	2.867	2.833
C8	7 Days	146.12	148.5	14.1471	2.067	2.101	2.084
	28 Days	218.34	216.77	14.1471	3.089	3.067	3.078
C9	7 Days	157.08	186.6	14.1471	2.222	2.640	2.431
	28 Days	211.78	210.5	14.1471	2.996	2.978	2.987
C10	7 Days	190.68	183.08	14.1471	2.698	2.590	2.644
	28 Days	243.97	248	14.1471	3.451	3.508	3.480
C11	7 Days	157.08	153.62	14.1471	2.222	2.173	2.198
	28 Days	163.99	161.79	14.1471	2.320	2.289	2.304
C12	7 Days	149.23	156.45	14.1471	2.111	2.213	2.162
	28 Days	190.29	195.8	14.1471	2.692	2.770	2.731
C13	7 Days	180	184.32	14.1471	2.546	2.608	2.577
	28 Days	184.73	178.76	14.1471	2.613	2.529	2.571
C14	7 Days	157.9	157.5	14.1471	2.234	2.228	2.231
	28 Days	208.3	206.34	14.1471	2.947	2.919	2.933
C15	7 Days	161.2	162.12	14.1471	2.281	2.294	2.287
	28 Days	198.58	196.7	14.1471	2.809	2.783	2.796

4.2. Test of Adequacy of the Model

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 − 95 = 5% significance. Since it is a two-tailed, significance = 5/2 = 2.5%

Hence significance level = 100 – 2.5 = 97.5%

Let D be difference between the experimental and predicted responses

The mean of the difference, $D_a = \frac{1}{n}\sum_{i=1}^{n} D_i$ (25)

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D-D_a)^2{}_i \qquad (26)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \qquad (27)$$

Where n = number of observations with degree of freedom n – 1.

$$S^2 = \frac{0.266}{15-1}; or\ S = \sqrt{0.019} = 0.138$$

$$t_{calculated} = 0.344$$

Table 5: Experimental and predicted values of 28days Split Tensile strength of Concrete

Sample Points	Response Y	Pseudo Components					Split tensile strength	Split tensile strength
		w-c ratio	Cement	Sand	SDA	Granite		
		X_1	X_2	X_3	X_4	X_5	$Y_{exp}(N/mm^2)$	$Y_{pred}(N/mm^2)$
BRE12	Y_1	1	0	0	0	0	2.987	2.987
BRE22	Y_2	0	1	0	0	0	2.378	2.378
USBR22	Y_3	0	0	1	0	0	2.625	2.625
BIS12	Y_4	0	0	0	1	0	3.799	3.799
ACI12	Y_5	0	0	0	0	1	3.176	3.176
N1	Y_{12}	0.5	0.5	0	0	0	3.069	3.069
N2	Y_{13}	0.5	0	0.5	0	0	2.222	2.222
N3	Y_{14}	0.5	0	0	0.5	0	2.684	2.684
N4	Y_{15}	0.5	0	0	0	0.5	3.060	3.060
N5	Y_{23}	0	0.5	0.5	0	0	2.856	2.856
N6	Y_{24}	0	0.5	0	0.5	0	2.922	2.922
N7	Y_{25}	0	0.5	0	0	0.5	2.867	2.867
N8	Y_{34}	0	0	0.5	0.5	0	3.036	3.036
N9	Y_{35}	0	0	0.5	0	0.5	2.353	2.353
N10	Y_{45}	0	0	0	0.5	0.5	2.809	2.809
C1	Y_{C1}	0.4	0	0.4	0	0.2	2.422	2.324
C2	Y_{C2}	0	0.6	0	0.4	0	2.682	2.787
C3	Y_{C3}	0.8	0	0.2	0	0	2.584	2.541
C4	Y_{C4}	0	0.4	0	0.6	0	3.158	3.071
C5	Y_{C5}	0.6	0	0	0	0.4	2.864	3.042
C6	Y_{C6}	0	0	0.8	0.2	0	2.847	2.747
C7	Y_{C7}	0.6	0.2	0	0	0.2	2.833	3.093
C8	Y_{C8}	0	0.4	0	0.4	0.2	3.078	2.811
C9	Y_{C9}	0.2	0	0	0	0.8	2.987	3.124
C10	Y_{C10}	0	0	0.2	0.8	0	3.480	3.452
C11	Y_{C11}	0.2	0	0.6	0	0.2	2.304	2.261
C12	Y_{C12}	0	0.2	0.4	0.2	0.2	2.731	2.682
C13	Y_{C13}	0	0.8	0.2	0	0	2.571	2.654
C14	Y_{C14}	0	0.2	0	0.2	0.6	2.933	2.832
C15	Y_{C15}	0.2	0.2	0.2	0.2	0.2	2.796	2.665

Table 6: Student t-test for 28days Split tensile strength of Concrete

Sample	Curing	Split tensile Strength (N/mm²)		t-test		
		$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D_a-D)^2$
C1	28 Days	2.422	2.324	0.098	-0.086	0.007
C2	28 Days	2.682	2.787	-0.105	0.117	0.014
C3	28 Days	2.584	2.541	0.043	-0.031	0.001
C4	28 Days	3.158	3.071	0.087	-0.075	0.006
C5	28 Days	2.864	3.042	-0.178	0.190	0.036
C6	28 Days	2.847	2.747	0.100	-0.088	0.008
C7	28 Days	2.833	3.093	-0.260	0.272	0.074
C8	28 Days	3.078	2.811	0.267	-0.255	0.065
C9	28 Days	2.987	3.124	-0.137	0.149	0.022
C10	28 Days	3.480	3.452	0.028	-0.016	0.000
C11	28 Days	2.304	2.261	0.043	-0.031	0.001
C12	28 Days	2.731	2.682	0.049	-0.037	0.001
C13	28 Days	2.571	2.654	-0.083	0.095	0.009
C14	28 Days	2.933	2.832	0.101	-0.089	0.008
C15	28 Days	2.796	2.665	0.131	-0.119	0.014
TOTAL				0.184		0.266
AVERAGE D_a				0.012		

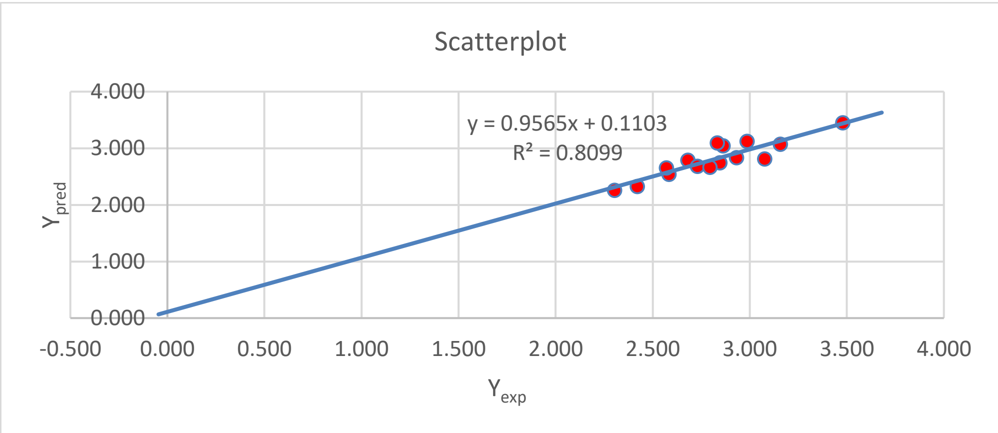

Fig. 4: Scatterplot of Predicted vs. Experimental 28days Split Tensile Strength

From the t-table, $t_{(\beta,v)}$ can be determined where $v = 15 - 1 = 14$, and β = significance level. $t_{(0.975,14)} = 2.145$

Since $t_{calculated}$ (0.344) < $t_{(0.975,14)}$ (2.145), and lies between -2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

The R^2 value of 0.8099 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

5. CONCLUSIONS AND RECOMMENDATIONS

Replacement of fine aggregate with 5% SDA has resulted in acceptable 28 days Split Tensile strengths (between 2.2 and 3.8N/mm²) with concrete mix ratios resulting from different design methods. A regression model has been generated from the resulting laboratory experiments using Scheffe's simplex theory. A two-tailed t-test was carried out, which confirmed the adequacy of the derived model with an R^2 value of 0.8099. The results also confirmed that SDA is a suitable material to replace a small fraction of fine aggregate in a bid to promote sustainability.

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