



THE USE OF DIMENSIONAL ANALYSIS FOR MODELING TRACTOR FUEL CONSUMPTION FOR HARROWING OPERATION

R. A, Ekemube¹, A. T, Atta^{2,*}, and A. Okogbule-Wonodi¹

¹Department of Agricultural and Environmental Engineering, Rivers State University, Port Harcourt, Nigeria

²Kenaf and Jute Improvement Program, Institute of Agricultural Research and Training, Obafemi Awolowo University, Moore Plantation, Ibadan, Nigeria

*corresponding author (Email:adekunleatta@gmail.com)

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Abstract

Fuel consumption model for predicting tractor fuel consumption per working area for harrowing operation has been developed using dimensional analysis. Generalized reduced gradient method of Excel solver was adopted for the establishment of the model's constant. The model was validated by simulating the experimental results into the equation and coefficient determination (r^2). Graphical comparison, root mean square error, and paired t-test were also used to validate. The field experiment was carried out at Rivers Institute of Agricultural Research and Training Farm in Rivers State University, Port Harcourt. The group balanced block design was adopted. The design consisted of 9 experimental treatments with three replicates. The experimental fuel consumption per working area was determined by quantity of fuel used per working area with the aid of fuel flow meter. The field test parameters were measured accordingly with their specific standard procedures. The field test parameters including bulk density, tractor forward speed, harrowing depth, and cone index were simulated with fuel consumption to obtain the constants in the prediction model. The developed model showed good agreement between measured and predicted results with high coefficient of determination and low root mean square error. The paired t-test results showed no significant difference at 95 and 99 % confidence levels. Thus, suggested that the model could be used for predicting tractor fuel consumption for harrowing operation.

Keywords: Dimension analysis, Fuel consumption, Harrowing, Model, Soil-machine-implement Parameters.

1.0 INTRODUCTION

Harrowing is a secondary tillage operation that involves the use of implement such as harrow for breaking the soil into smaller particles for preparing seedbed conducive for crop production. This can be done after ploughing operation. It has been reported by [1] that tractor use more energy at the lowest work rate during harrowing operation. Tractor fuel consumption is affected by many parameters during tillage operation, these include type and structure of soil, climate, tractor type, tractor size and tractor-implement relationship [2 - 5]. [6] studied tractor performance monitors optimizing tractor and implement dynamics in tillage operations for one year of field tests. They concluded that the 24 disc implements performed better both in terms of work rate and fuel consumption per unit of worked area, though by a small difference, relative to the 20-disc

harrow, making the larger implement a better choice. Correia et al. [7] assumed that engine rotation speed and the effective work speed may be decision making questions in the harrowing operation. They found that the increase in effective speed reduces harrow work depth, from 3.65 km/h to 2.5 km/h (157 rad/s, 1500 rpm); the depth is reduced in approximately 26%, going from 17.7 to 13.1 cm. This resulted in tillage with compromised value, being only rapidly equipped.

The estimation of tractor fuel consumption during harrowing operation has been determined by different methods. These methods are usually focused on supplies of power and individual engines, which call for extensive engine testing to validate the amount of fuel consumed [8 - 12]. Few models have been developed by various researcher to predicted tractor

fuel consumption per working area during harrowing operation [13] developed fuel consumption model in litres per hectare (l/ha) using multiple linear regression method during harrowing Operation. [14] used dimensional analysis to develop tractor fuel consumption model during harrowing operation. This model by Nkakini et al. [14] is for predicting fuel consumption per working hour. But, in literature there is a dearth of information on tractor fuel consumption model for predicting fuel consumption per working hour for harrowing operation. Therefore, there is need to develop a fuel consumption model in terms of working area using dimensional analysis. The aim of this study is to develop a prognostic model for predicting tractor fuel consumption per working area for harrowing operation.

2.0 MATERIALS AND METHODS

2.1 EXPERIMENTAL SITE DESCRIPTION

This experiment was performed at the Rivers Institute of Agricultural Research and Training (RIART) farm at Rivers State University, Port Harcourt, Nigeria (latitude of $4^{\circ} 49' 27''$ N, and longitude of $7^{\circ} 2' 1''$ E). The experimental design used in this study is Group Balanced Block Design (GBBD). A farm size of 138 m by 50 m (6900 m^2) was divided into three plots of 9 sub-plots each. Each sub-plot of 50m by 2m was marked with a 1m alley. The sub-plot was provided for different treatment options and with a space of 2 m between each block and 1 m at the sides of the outer blocks.

2.2 TRACTOR AND IMPLEMENT SPECIFICATIONS

The tractor used to carry out harrowing operation in this study has a total weight of 3015kg, engine horsepower of 72 hp and lifting power of 2200 kg. Front and the rear tyres were 7.5 –16, 8 ply and 16.9 – 28, 12 radials respectively (Figure 1). A 1800 mm frame width mounted-type disc harrow with disc diameter of 508 mm of disc harrow (Baldan Implementos Agricolas, Brazil) with 9-disc bottom mounted on a gauge wheel was used for the experiments (Figure 2).

2.3 FUEL FLOW METER SPECIFICATION

The DFM 100CD fuel flow meter (Technoton Engineering, Belarus) has nominal fuel pressure 0.2 MPa, maximum fuel pressure 2.5 MPa, minimum kinematic viscosity $1.5 \text{ mm}^2/\text{s}$, maximum kinematic viscosity $6.0 \text{ mm}^2/\text{s}$, minimum supply voltage 10 V and maximum supply voltage 45 V (Figure 3).



Figure 1: The Swaraj 978 FE Tractor (Swaraj, India)



Figure 2: The Disc Harrow (Baldan Implementos Agricolas, Brazil) used in this Study



Figure 3: DFM 100CD Fuel Flow Meter (Technoton Engineering, Belarus) used in this Study (Source: Technoton Fuel Flow Meter Manual)

2.4 MODEL DERIVATION

The significance of accurate prediction in any field of engineering cannot be eliminated. Therefore, mathematical tool that was employed in this study is dimensional analysis using Buckingham pi theorem. Hence, in this research fuel consumption model development were derived using method of fuel consumption per working area (FC_{twa} , L/ha). Some of the factors influencing tractor fuel consumption were

presented in Table 1 and the dimensional matrix in Table 2.

Table 1: Dimensions of Some Variables Influencing Fuel Consumption

Variables	Symbol	Unit	Dimensions
Dependent Variable			
Fuel consumption	FC _{wa}	L/ha	L ³ L ⁻² (L)
Independent Variables			
Forward speed	V	Km/h	LT ⁻¹
Harrowing depth	d	m	L
Cone index	CI	N/cm ²	ML ⁻¹ T ⁻²
Bulk density	ρ _b	g/cm ³	ML ⁻³
Width of cut	W	m	L

Table 2: Dimensional Matrix of the Variables

Dimensions	Parameters					
	FC _{wa}	V	d	CI	ρ _b	W
M	0	0	0	1	1	0
L	1	1	1	-1	-3	1
T	0	-1	0	-2	0	0

Fuel consumption, FC_{wa} is a function of (*d*, *W*, *V*, *CI*, ρ_b)

Mathematically;

$$FC_{wa} = f(d, W, V, CI, \rho_b) \quad (1)$$

The dependent variable = FC_{ta}

Total number of variables, *n* = 6

Total number of fundamental dimensions, *m* = 3

Therefore, number of dimensionless groups (*π*- terms) to be formed = *n* – *m* = 6 – 3 = 3

Equation 1 can be written as:

$$f(\pi_1, \pi_2, \pi_3) \quad (2)$$

Each *π*- term contains (*m* + 1) variables, where *m* = 3 and is also equal to repeating variable choosing from ρ_b, *W*, *V* as repeating variables, gives five *π*- terms as:

$$\pi_1 = \rho_b^{a_1} \cdot W^{b_1} \cdot V^{c_1} \cdot FC_{wa} \quad (3)$$

$$\pi_2 = \rho_b^{a_2} \cdot W^{b_2} V^{c_2} \cdot d \quad (4)$$

$$\pi_3 = \rho_b^{a_4} \cdot W^{b_4} V^{c_4} \cdot CI \quad (5)$$

2.4.1 Transformation to Dimensionless Parameters

A new set of *pi* terms can be generated by multiplication or division with each other. In addition, the present *pi* terms can be reversed to make a new *pi* term. This is to ensure simplicity in the experimentation process. The present *pi* terms

(*π*₁, *π*₂, and *π*₃) can be adjusted to generate a new *pi* term [14 – 18].

*π*₁ terms

$$\pi_1 = \frac{FC_{wa}}{d^2 V} \quad (6)$$

*π*₂ – terms

$$\pi_2 = \frac{W}{d} \quad (7)$$

*π*₃ – Terms

$$\pi_3 = \frac{CI}{\rho_b V^2} \quad (8)$$

Substituting the values of *π*₁, *π*₂, *π*₃, *π*₄ and *π*₅ in equation (2):

$$f\left(\frac{FC_{wa}}{W}, \frac{d}{W}, \frac{CI}{\rho_b V^2}\right) = 0 \quad (9)$$

Where:

f = functional notation for fuel consumption

2.4.2 Formulation of the Fuel Consumption Model

The method of product and quotient component functions were adopted for development of the fuel consumption model. This prognostic model was developed by simple multiplication and division of the component equations. The validity of combining the equation components by multiplication and division were tested by assuming that the general prediction model is obtained by simple multiplication and division of the *pi* terms (Equations 10 and 11).

Dividing Equation 7 by Equation 8 to establish *π*₁¹:

$$\pi_1^1 = \frac{\pi_2}{\pi_3} = \frac{\frac{d}{W}}{\frac{CI}{\rho_b V^2}} \quad (10)$$

$$\pi_1^1 = \frac{\rho_b V^2 d}{CIW} \quad (11)$$

Hence, the relationship becomes

$$\pi_1 = K_{FC} f(\pi_1^1) \quad (12)$$

$$\frac{\pi_1}{\pi_1^1} = K_{FC} \quad (13)$$

Substituting the values of *π*₁ and *π*₁¹ into Equation (12):

$$\frac{FC_{wa}}{W} = K_{FC} \left[\frac{\rho_b V^2 d}{CIW} \right] \quad (14)$$

*K*_{FC} can be calculated using method of GRG in excel solver and the constant obtained becomes the *K*_{FC} value

$$\therefore K_{FC} = \frac{FC_{wa}CI}{\rho_b V^2 d} \quad (15)$$

Rearranging equation (15), it becomes:

$$FC_{wa} = K_{FC} \left[\frac{\rho_b V^2 d}{CI} \right] \quad (16)$$

Equation (16) expresses the tractor fuel consumption per working area during ridging operation.

Where:

K_{FC} = Fuel consumption constants

2.4.3 Model Validation

The developed model was validated by simulating the experimental data and then comparing the experimental with the prediction data. The parameters (ridge height, forward speed, cone index and bulk density) were determined experimentally and substituted into the formulated model to compute the predicted fuel consumption per working area.

Also, the root mean square error (RMSE) was used to check the error difference as represented in Equation (17).

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (FC_{wa(m)} - FC_{wa(P)})^2}{N}} \quad (17)$$

Where:

N = number of samples,

$FC_{wa(m)}$ = measured fuel consumption (L/ha)

$FC_{wa(P)}$ = predicted fuel consumption (L/ha).

Furthermore, the developed model was validated with regression curve and coefficient of determination (r^2). These were used to check if the measured and predicted results have good agreement. Also, graphical comparison of measured and predicted results as well the paired t-test as presented in Equation (18) was considered as significant at $t_{computed} > t_{table}$ (95 and 99 % confidence) levels.

$$t = \frac{\sum D/N}{\sqrt{\frac{\sum D^2 - \left(\frac{(\sum D)^2}{N}\right)}{(N-1)(N)}}} \quad (18)$$

Where:

$\sum D$ = summation of the differences.

$\sum D^2$ = summation of the squared differences,

$(\sum D)^2$ = summation of the differences squared.

2.5 EXPERIMENTAL PROCEDURES

Prior to harrowing operation, soil core was used for obtaining the soil sample from the depth of 0 to 10, 10 to 13 and 13 to 16 cm respectively at random in the field to determined textural classification of the soil and the bulk density. The collected soil samples were taken to the laboratory for analysis. The bulk density was determined using core method [19].

The disc harrow was attached to the tractor and levelled using the top links of the tractor in order to reduce parasitic forces. The harrowing depths were determined by setting the level control of the lifting mechanism (three-point linkage height) to lower the disc harrow to the desired harrowed depth. Tractor forward speeds were determined by selecting a particular gear that gave the desired speed. This was done in a practice area in advance for each test plot to maintain the desired treatment. The harrowing depth measurement was done by placing the meter rule from furrow bottom to the surface of the harrowed land, while the width of cut was measured by placing a steel tape from one side of the furrow wall to the other end. Time was determined with a stopwatch set at zero before each operation. The cone index was also determined using a cone penetrometer.

The digital method of measuring the quantity of fuel used was adopted to determine tractor fuel consumption. During this process, the use of DFM fuel flow meter was employed to measure fuel consumption. The metre was mounted on the fuel line between the tractor's fuel tank and the pump. At the end of each test operation, the data was taken from the fuel flow meter as display information, switching is performed by light touch to the top cover of fuel flow meter by iButton key. Mathematically, fuel consumption per working area was calculated by expression in equation 19 [20]:

$$FC = \frac{10 \times T_{fc}}{V \times W \times E \times h} \quad (19)$$

Where:

FC = Fuel consumption per working area, (L/ha)

T_{fc} = Tractor fuel consumption, (L)

V = Forward speed, (Km/h)

W = Implement width, (m)

E = Implement field efficiency, (%)

h = Working hour, (h)

3.0 RESULTS AND DISCUSSION

3.1 ESTABLISHMENT OF CONSTANT(K_{FC}) FOR FUEL CONSUMPTION PER WORKING AREA

From Equation 16, K_{FC} was constant for the fuel consumption per working area model formulated using fuel flow meter. The generalized reduced gradient method of excel solver was used to compute the constants by simulating measured field test results CI , d , measured FC_{wa} , predicted FC_{wa} and error squared and the values for the constants (K_{FC}) was established for harrowing operation (Table 3). It is represented as:

$$K_{FC} = 181.5027$$

The computed constants (K_{FC}) of the model formulated were fitted into the fuel consumption per

working area model established. The results showed acceptable agreement with minimum error, revealing the reliability and acceptability of the model. Therefore, consumption model per working area established for harrowing operation at tractor forward speed, harrowing depth, width of operation, cone index, bulk density, respectively:

$$FC_{wa} = 181.5027 \left[\frac{\rho_b V^2 d}{CI} \right] \quad (20)$$

This model is similar to that of [21] that used multiple linear regression analysis with excel spread sheet and fitted to the model structure formulae to calculate the coefficient. Also, equation 20 is similar to fuel consumption equations developed by [14, 17 and 18] that used linear regression to fit in their fuel consumption equations to determined constants.

Table 3: Fuel Consumption per Working Area and Operating Conditions for Harrowing

Treatment	Parameters						(Error) ²
	ρ_b (g/cm ³)	V (km/h)	CI (N/cm ²)	d (m)	Measured FC _{wa} (L/ha)	Predicted FC _{wa} (L/ha)	
1	3818.88	5.00	164.06	0.10	3.15	3.9274	0.6044
2	4133.38	7.00	164.06	0.10	6.67	7.6978	1.056303
3	4447.87	9.00	164.06	0.10	12.8	12.7249	0.005643
4	5346.43	5.00	214.48	0.13	4.86	4.2009	0.434409
5	5786.73	7.00	214.48	0.13	9.61	8.2338	1.89401
6	6227.02	9.00	214.48	0.13	13.85	13.6109	0.057157
7	6873.98	5.00	253.91	0.16	4.33	4.7179	0.150453
8	7440.08	7.00	253.91	0.16	9.52	9.2471	0.074502
9	8006.17	9.00	253.91	0.16	14.76	15.2859	0.276613

$$K_{FC} = 181.5027 \text{ and } SS_E = 4.553489$$

3.2 VALIDATION OF THE MODEL

The actuality of a formulated model for solving a particular problem depends on its estimates and validation. Results of the formulated fuel consumption per working area model for harrowing operation was by substitution of the results of a number of measured data which is being compared with the predicted fuel consumption per working area. Figure 4 and 5 showed the graphical comparison between measured and predicted fuel consumption per working area values using fuel flow meter. It was observed that the model has a high relationship with measured data from the harrowing operation with coefficient of determination (r^2) value of 0.964.

Based on comparing the means of estimated and measured data statistically, it was revealed that the root mean square error (RMSE) analysis which illustrated the error differences between the measured and predicted results is 1.91. Additionally, the t-test

was used to determine the level of significance between the means of measured and predicted fuel consumption per working area at 0.05 and 0.01 significance levels. The value of the paired t-test ($t_{calculated}$) was (0.52) that is less than t_{table} values (2.306 and 3.355) (i.e., $t_{cal} < t_{tab}$).

This pointed out that there is no significant difference between the measured and the estimated data. This was comparable to the finding of [22] that developed fuel consumption model of a chisel plough using dimensional analysis. Also, ASAE (2000a) referenced ASAE EP496 that the fuel consumption equations developed was 15 % higher than the Nebraska tractor test performance under field condition. As well, [14, 17 and 18] used dimensional analysis in Buckingham pi theorem to develop fuel consumption model for ploughing, harrowing and ridging, respectively.

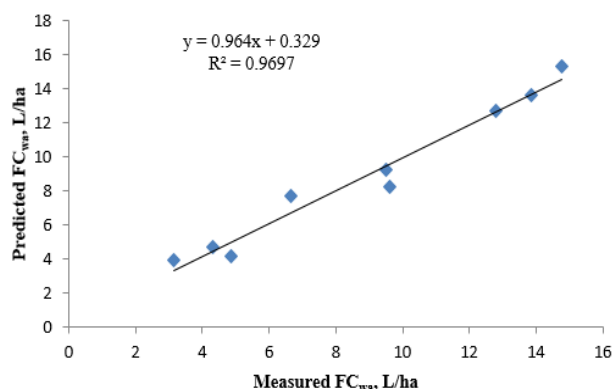


Figure 4: Predicted vs Measured FC_{wa} for Harrowing

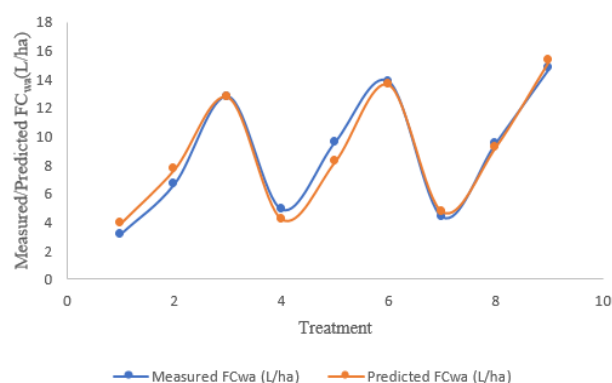


Figure 5: Measured and Predicted FC_{wa} vs Treatment for Harrowing

4.0 CONCLUSION

This study had developed a tractor fuel consumption predictive model for harrowing operation to ensure prediction of fuel consumption per working area usage using dimensional analysis. The following conclusions were drawn from the results obtained:

- i. A prognostic model for predicting tractor fuel consumption per working area in course of harrowing operation has been developed using dimensional analysis.
- ii. The model developed has a constant (K_{FC}) of 181.5027 for harrowing operation with respect to the equipment used.
- iii. Model prediction achieved in this study can be categorized as good for high coefficient of determination (r^2), low root mean square error, and paired t Test computed was less than the table value.
- iv. Hence, these results displayed acceptable agreement with measured and predicted model results proving that the model can predict experimental data accurately.

REFERENCES

- [1] Ikpo, I. and Ifem, J. L. C. (2005). "Fuel Consumption of Tractors – Steyr 768 & 8075 during Tillage Operations in Makurdi, Benue State", Proceedings of the Nigerian Institution of Agricultural Engineers, 2005, 27, pp.60 – 63.
- [2] Fathollahzadeh, H., Mobli, H., Rajabipour, A., Minaee, S., Jafari, A. and Tabatabaie S.M.H. "Average and Instantaneous Fuel Consumption of Iranian Conventional Tractor with Moldboard plow in tillage", Asian Research Publishing Network, Journal of Engineering and Applied Sciences, 2010, 5(2), 30-35.
- [3] Ajav, E.A. and Adewoyin, A. O. "Effect of Ploughing Depth and Speed on Tractor Fuel Consumption in a Sandy-Loam Soil of Oyo State", Nigeria. Journal of Agricultural Engineering and Technology, 2012, 20(2), pp.1-10.
- [4] Adewoyin, A. O. "Fuel Consumption Evaluation of Some Commonly Used Farm Tractors for Ploughing Operations on the Sandy-Loam Soil of Oyo state, Nigeria", Research Journal of Applied Sciences, Engineering and Technology, 2013, 6(15), pp.2865-2871.
- [5] Adewoyin, A. O. and Ajav E. A. "Fuel Consumption of Some Tractor Models for Ploughing Operations in the Sandy-Loam Soil of Nigeria at Various Speeds and Ploughing Depths", International Commission for Agricultural Engineering Journal, 2013, 15(3), pp.67-74.
- [6] Peça, J. O., Serrano, J. M., Pinheiro, A., Carvalho, M., Nunes, M., Ribeiro, L., and Santos, F. Tractor Performance Monitors Optimizing Tractor and Implement Dynamics in Tillage Operations - One Year of Field Tests. European Agricultural Engineering, 1988, Paper no: 98 - 131.
- [7] Correia, T. P.S., Sousa, S. F. G., Tavares, L.A. F., Silvap. R. A., and Riquetti, N. B. (2015). "Disk Harrow Operational Performance in Three Engine Rotation Speeds", Científica Jaboticabal, 2015, 43(3), pp.221-225.
- [8] ASAE. ASAE Standards, 49th Ed. 2002a. EP496.2. "Agricultural Machinery Management Data", St. Joseph, Michigan: ASAE, 2002a.
- [9] ASAE. ASAE Standards, 49th Ed. 2002b. D497.4 JAN98. "Agricultural Machinery Management Data", St. Joseph, Michigan: ASAE, 2002b.
- [10] Grisso, R. D., Kocher, M. F. and Vaughan, D. H. "Predicting Tractor Fuel Consumption.

- Applied Engineering in Agriculture”, 2004 20(5), pp.553-561.
- [11] Grisso, R. B., Perumpral, J. V. and Vaughan, D., Roberson, G. T., and Pitman, R. “Predicting Tractor Diesel Fuel Consumption”, Virginia Cooperative Extension Publication VCE publication, 2010, pp.442-073. <http://pubs.ext.vt.edu.edu/442-073> (accessed on December 5th, 2016).
- [12] Grisso, R. B., Perumpral, J. V., Roberson, G. T. and Pitman, R (2011). Predicting Tractor Diesel Fuel Consumption. Virginia Cooperative Extension Publication VCE publication, 2011, pp.442-073. <http://pubs.ext.vt.edu.edu/442-073> (accessed on September 5th, 2016)
- [13] Oyelade, O. A. and Oni, K. C. “Modelling of Tractor Fuel Consumption for Harrowing Operation in A Sandy Loam Soil”, Arid Zone Journal of Engineering, Technology and Environment, 2018, 14(I4), pp.8-19.
- [14] Nkakini, S. O., Ekemube, R. A. and Igoni, A. H. “Modeling Fuel Consumption Rate for Harrowing Operations in Loamy Sand Soil”, European Journal of Agriculture and Forestry Research, 2019b, 7(2), 1 – 12.
- [15] Langhaar, H. L. “Dimensional Analysis and Theory of Models”, New York: Wiley, 1980.
- [16] Tarham, S. and Carman, K. “Modeling the Torque and Power Requirement of Traction Tires of Horticultural Tractors using Dimensional Analysis”, Mathematical and Computational Applications, 2004, 9(3), 427 - 434.
- [17] Nkakini, S. O., Ekemube, R. A. and Igoni, A. H. “Development of Predictive Model for Fuel Consumption During Ploughing Operation in Agricultural Soil”, European Journal of Engineering and Technology, 2019a, 7(1), 16 – 30.
- [18] Igoni, A. H., Ekemube, R. A. and Nkakini, S. O. “Predicting Tractor Fuel Consumption during Ridging on a Sandy Loam Soil in a Humid Tropical Climate”, Journal of Engineering and Technology Research, 2019, 11(3), 29-40.
- [19] Walter K., Don, A., Tiemeyer. B. and Freibauer, A. “Determining Soil Bulk Density for Carbon Stock Calculations: A Systematic Method Comparison”, Soil Science Society American Journal, 2016, 80: 579–591
- [20] Shafaei, S. M., Loghari, M. S., and Kamgar, S. “On the Neurocomputing Based Intelligent Simulation of Tractor Fuel Efficiency Parameters”, Information Processing in Agriculture, 5, 2018, 205 – 223. (Accessed on July 6th, 2018) (Available at www.sciencedirect.com)
- [21] Kumar, N. and Pandey, K. P. “A Visual Basic Program for Predicting Optimum Gear and Throttle Position for Best Fuel Economy for 32 Kw Tractor”, Computer and Electronic in Agriculture, 119, 2015, 217-257.
- [22] Karparvarfard, S. H., and Rahmanian-Koushkaki, H. Development of a Fuel Consumption Equation: Test Case for a Tractor Chisel-Ploughing in a Clay Loam Soil. Biosystems Engineering, 2015, 130, 23 – 33. DOI: 10.1016/j.Biosystemseng.2014.11.015.