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FAILURE INVESTIGATION OF REINFORCED CONCRETE CANTILEVER RETAINING WALLS

I. Abubakar[a], **H.B. Peter**[b]

Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.
Emails: [a]idrcivil1@yahoo.com; [b]l_net29@yahoo.com

Abstract

Failure investigation was carried out on a reinforced concrete cantilever retaining wall using First Order Reliability Method (FORM). Four failure modes of the retaining wall were considered namely: overturning; sliding; bearing capacity and bending moment failure modes. The results of the analysis generally showed that the bending moment mode of failure was the most critical, followed by the sliding, the overturning, and by the bearing capacity modes of failure. Also, as the magnitude of the design load was increased by 50kN/m^2, the safety of the retaining wall increased by about 5%, 6% and 25% considering overturning, sliding and bearing capacity failure modes respectively. However, the safety of the structure increased by about 18% when the magnitude of steel reinforcement ratio was increased by 0.1%. Again, when the width of backfill was increased by about 15%, the safety of the structure decreased by about 25% and 34% considering overturning and bearing capacity failure modes; and increased by about 33% and 4% considering sliding, and bending moment failure modes respectively.

Keywords: failure investigation, safety index, cantilever retaining walls

1. Introduction

The aim of a design is to produce structural members with good safety margin, so that the risk of failure is small. While deterministic design provides output that is precise and exact, it however ignores the uncertainties of the input design variables. These uncertainties must be taken into account to assess the safety and performance of the structure by the use of the concept of structural reliability [1].

Many sources of uncertainties are inherent in structural design. Despite what designers often think, the parameters of the loadings and the load carrying capacities of structural members are not deterministic quantities. They are random variables, and thus absolute safety or zero probability of failure cannot be achieved [2].

Engineering design decisions are surrounded by uncertainties that result from the random nature of loading and structural resistance as well as the load and resistance prediction models. The effect of such uncertainties is included in design through the use of safety factors that are based on engineering judgment and previous experience with similar structure. Underestimation of these uncertainties sometimes leads to adverse results of collapse such as those reported by Lew et al [3]. In general, because of uncertainties, the question of safety and performance has arisen [4]. Due to the fact that safety involves a consideration of random variables and the realization of the limitations in design by the deterministic method, it is now generally accepted that the rational approach to the analysis of safety is through the use of probabilistic models [4, 5].

The design of safe structures is generally conducted by checking the performance of individual structural elements, with regard to the actual role in preventing potential collapse of the entire structure [6]. It is in line with this that there is the need to evaluate the design criteria of reinforced cantilever retaining walls using reliability concepts.

The work presented in this paper therefore focused on the failure investigation of a reinforced concrete cantilever retaining wall designed to the requirements of BS8110 [7] and BS8004 [8]. This was achieved by the use of FORM. Four failure criteria for a safe design of the wall were considered, which include overturning, sliding, bearing capacity and bending moment failure criteria. Effect of the variations of basic decision variables was also examined, and the corresponding safety indices computed by the use of FORM.

2. First Order Reliability Method

In the evaluation of safety using probabilistic concepts it is often the practice to define a safety margin Z on the basis of the performance function $g(x_1,\ldots,x_n)$ which relates the resistance of the component to the applied loading. That is,

$$Z = g(x_1,\ldots,x_n) \tag{1}$$

In equation (1) $g(x_1,x_2,\ldots,x_n)$ represent the design variables in the design equations. Since the individual members of this function may be random quantities Z also must be a random variable which must satisfy the condition that $Z > 0$ at the internal points of the safe set, $Z = 0$ at the limit state, and $Z < 0$ at the internal points of the failure set. A generalized simple safety index can be formed if the random variables collected in the vector X are normalized and collected in another vector Y using a linear mapping of the kind $X = L(Y)$ such that $Y = L - 1(X)$. Therefore the corresponding space of points is then defined by the transformation

$$x = L(y), \quad y = L^{-1}(x) \tag{2}$$

The consequence of this transformation maps equation (1) at the limit state into

$$h(y_1,\ldots,y_n) = 0 \tag{3}$$

in which

$$h(y) = g(L(y)) \tag{4}$$

The mean value of Y occurs at the origin while the projection of Y on a straight line through the origin is a random variable with a unit standard deviation. The distance from the origin to the limit state surface in this normalized space becomes the geometric safety index. In other words,

$$\beta = m\,in\left\{\sqrt{y'y}|(y)=0\right\} \tag{5}$$

where the minimum of the distance β from the origin to y is obtained for varying y over the entire limit state surface. A point y on this limit surface that actually corresponds properly to the globally most central limit-state point is the checkpoint corresponding to the sought probable failure point.

2.1. Performance functions

The calculation of the performance function is performed for discrete combination of basic variables into the failure modes of reinforced concrete cantilever retaining walls in accordance with BS8110 [7] and BS8004[8] given as follows:

2.1.1. Overturning mode of failure

The performance function considering overturning failure mode of the retaining wall is given by,

$$G(x) = \left[\left(\gamma_{conc}Ht_s\left(\frac{t_s}{2}+B-t_s-w_s\right)\right)+\left(0.5\gamma_{conc}t_bB^2\right) + \left(w_sH\gamma_{fill}\left(\frac{w_s}{2}+B-w_s\right)\right)+\left(\alpha\left(\frac{t_s}{2}+B-t_s-w_s\right)\right)\right] - 1.78\left[\frac{0.5K_a\gamma_{fill}\left(H+t_b\right)^3}{3}+0.5K_a(H+t_b)^2q_s\right]\leq 0 \tag{6}$$

Where γ_{conc}, q_s, and γ_{fill} are the unit weights of concrete, surcharge and backfill respectively. t_s and t_b are the thicknesses of stem and base of the wall, B and w_s are widths of base and backfill respectively, H is the height of stem, α is the beam estimated loading and K_a is active earth pressure on the wall.

2.1.2. Sliding mode of failure

The performance function considering sliding failure mode of the retaining wall is given by,

$$G(x) = \left\{\left[\gamma_{conc}Ht_s+\gamma_{conc}t_bB+w_sH\gamma_{fill}+\alpha\right] -3.56\left[0.5K_a\gamma_{conc}(H+t_b)^2+K_a(H+t_b)q_s\right]\right\}\leq 0 \tag{7}$$

All the terms in equation (7) are as defined in section 2.1.1

2.1.3. Bearing capacity mode of failure

Also, the performance function considering base failure mode of the retaining wall is given by,

$$Gx = \left(\frac{N}{B}-\frac{6M}{B^2}\right)-\left((s_cd_ci_ccN_c)+(s_qd_qi_qq_oN_q) + \left(\frac{s_\gamma d_\gamma i_\gamma\overline{B}_\gamma N_\gamma}{2}\right)\right)\leq 0 \tag{8}$$

The following general bearing capacity equation for a strip foundation as attributed to Vesic [9] bearing capacity considerations, though Terzaghi [10], Meyerhof [11] and Hansen [12] also have some considerations as regards bearing capacity determination but for the purpose of this work all factor of bearing capacity considerations are all based on Vesic's equation given as follows:

$$\frac{Q}{B} = q_u = (s_cd_ci_ccN_c)+(s_qd_qi_qq_oN_q)+\left(\frac{s_\gamma d_\gamma i_\gamma\overline{B}_\gamma N_\gamma}{2}\right) \tag{9}$$

Where in equations (8) and (9), N and M are applied axial loading and moment, q_u is the ultimate bearing capacity, γ is the density of soil below foundation level, P_o is the effective overburden soil pressure at foundation level, Q is the normal component of the ultimate bearing capacity of the foundation, $\overline{B}$ is the

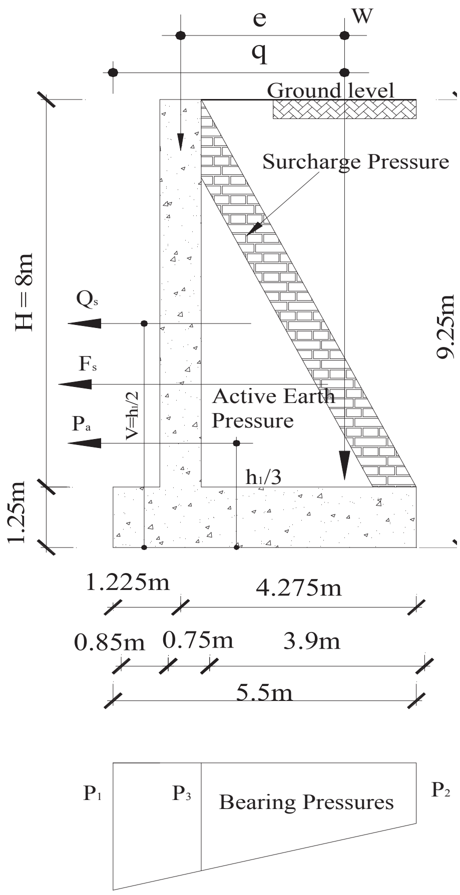

Figure 1: A cantilever retaining wall.

effective width of the base, c is the cohesion of the retained soil, s_γ, s_q and s_c are shape factors; d_γ, d_q and d_c are the depth factors; i_γ, i_q and i_c are the load inclination factors; and N_q, N_c, N_γ are the bearing capacity factors under vertical load.

2.1.4. Bending moment mode of failure

The performance function considering the bending failure of the base of the retaining wall is given by,

$$G(x) = \Big[1.35\left(\gamma_{fill} w_s H\right)\left(\frac{w_s}{2}\right) + 1.35\left(\gamma_{conc} t_b B\right)\left(\frac{w_s}{2}\right) - \frac{1}{2}\left(P_3 + P_2\right) w_s(x)\Big] - \frac{f_y \rho b d^2}{1.15}\left(1 - \frac{0.45 f_y \rho}{0.4623 f_{cu}}\right) \quad (10)$$

In equation (10), the terms P_1, P_2 and P_3 are bearing pressures as shown on the figure 1, f_y and f_{cu} are the characteristic strength of steel and concrete respectively, b and d are the width and effective depth of the section respectively, ρ is the reinforcement ratio of steel, while other terms are as defined earlier.

3. Results and Discussion

3.1. Example of a cantilever retaining wall

The cantilever retaining wall shown in Fig. 1 was designed as a retaining structure with load acting on the vertical wall as uniformly distributed. The back face of the abutment wall is subjected to hydrostatic force from ground water. The backfill material is a granular material of saturated density of 1960kg/m^3, and of cohesion and angle of internal friction of 15kN/m^2 and 30° respectively. The stem wall is cantilever wall of height, H of 8m, a base width, B of 5.5m, of stem thickness of wall, t is 750mm, of thickness of the base, t_b is 1.25m and the width of the granular material is 3.9m. The surcharge loads, q_s is equal to 10kN/m^2, the effective overburden soil pressure at foundation level, P_o is 10kN/m^2.

The characteristic strengths of steel and concrete are 410 N/mm^2 and 30N/mm^2 respectively. The retaining wall was designed in accordance with the design requirements of BS8110 [7] and BS8004 [8], by satisfying the failure modes stated in section (2.1). The design details were as tabulated in Table 1.

3.2. Stochastic models

The stochastic models consisting of the limit state functions given in the previous section were prepared in accordance with FORM [13].

3.3. Results

Reliability analyses of the wall designed in section (3.1) was achieved by the use of FORM [7] by estimating the reliability levels at varying heights of wall, widths of backfill, thicknesses of stem of wall, unit weights of soil, the bearing capacity factors, base slab thicknesses, reinforcement ratios; at varying values of applied loading.

Safety indices were obtained from the programs considering the failure modes of the walls as described in section (2.1). Plots of the safety indices versus the varied design variables were as shown in figures 2 to 22. The results generally showed that bending moment mode of failure is more sensitive than other modes considering variations in the design variables considered. This is followed by sliding mode of failure, by overturning mode of failure, and by bearing capacity mode of failure which has the highest range of reliability indices. This implies that the structure is safer considering the bearing capacity of the soil.

The results are further discussed against each failure mode.

3.3.1. Overturning mode of failure

The structure was checked for overturning moment and reliability results were estimated for different variables ranging from the height of wall, base slab thickness and width, width of backfill, stem thickness, and unit weight of soil. The results are as presented graphically in figures 2 to 7.

It was found out from the study that as the magnitude of the design load was increased by 50kN/m^2

Table 1: Experimental results.

S/No	Basic Variable	Variable Distribution Type	Mean	Coefficient of Variation	Standard Deviation
1	Stem thickness, t_s	Normal	0.75m	0.01	0.0075m
2	Base thickness, t_b	Normal	1.25m	0.01	0.0125m
3	Base width, B	Normal	5.5m	0.01	0.055m
4	Width of backfill, w_s	Normal	3.9m	0.01	0.039m
5	Stem Height, H	Normal	8m	0.01	0.08m
6	Surcharge load, q_s	Log normal	10kN/m^2	0.03	0.3kN/m^2
7	Unit weight of Concrete, ρ_{conc}	Log normal	24kN/m^3	0.06	1.44kN/m^3
8	Unit weight of Backfill, γ	Log normal	19kN/m^3	0.06	1.176kN/m^3
9	Beam estimates, α	Log normal	400kN/m	0.03	12kN/m
10	Length of Base	Log normal	15m	0.01	0.15
11	Beam estimates, α	Log normal	400kN/m	0.03	12kN/m
12	Depth of embedment, D_f	Normal	2m	0.01	0.02m
13	N_c	Normal	30.14	0.01	0.3014
14	N_q	Normal	18.40	0.01	0.184
15	N_γ	Normal	11.20	0.01	0.112
16	Cohesion	Normal	15kN/m^2	0.06	0.90 kN/m^2
17	Pore Pressure	Normal	10kN/m^2	0.06	0.6kN/m^2
18	Steel Strength, f_y	Normal	4.6x10^{5}kN/m^2	0.01	4.6x10^{3}kN/m^2
19	Concrete Strength, f_{cu}	Normal	3.0x10^{4}kN/m^2	0.01	3.0x10^{2}kN/m^2
20	Beam breadth, b	Normal	1.0m	0.01	0.01m
21	Rho(ρ)	Lognormal	0.0035	0.01	3.5x10^{-5}
22	Unit weight of Soil, γ_{soil}	Lognormal	19kN/m^3	0.06	1.14kN/m^3
23	Cover	Normal	0.04m	0.01	0.0004m
24	Diameter of Steel, ∅	Normal	0.016m	0.01	0.00016m
25	Bearing Pressure P_3	Normal	341.99kN/m^2	0.03	10.3kN/m^2
26	Bearing Pressure P_2	Normal	149.44kN/m^2	0.03	4.48kN/m^2

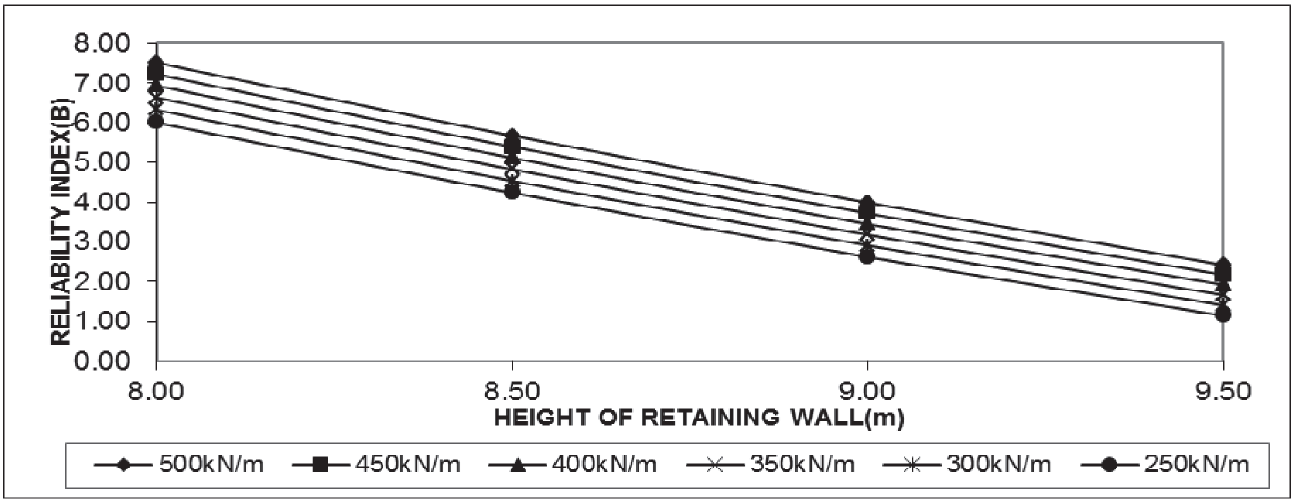

Figure 2: Reliability index versus height of retaining wall.

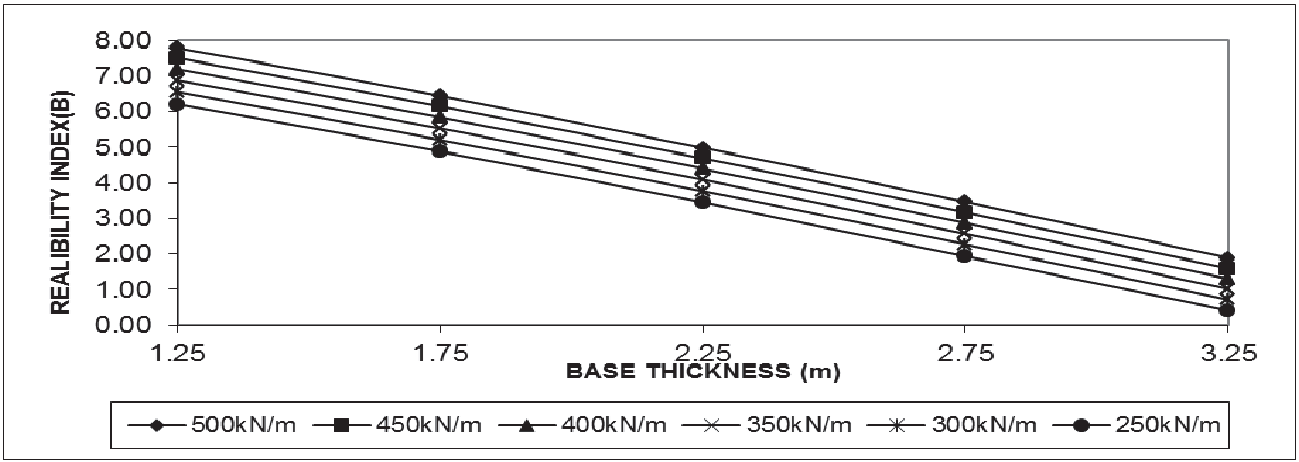

Figure 3: Reliability index versus base thickness of retaining wall.

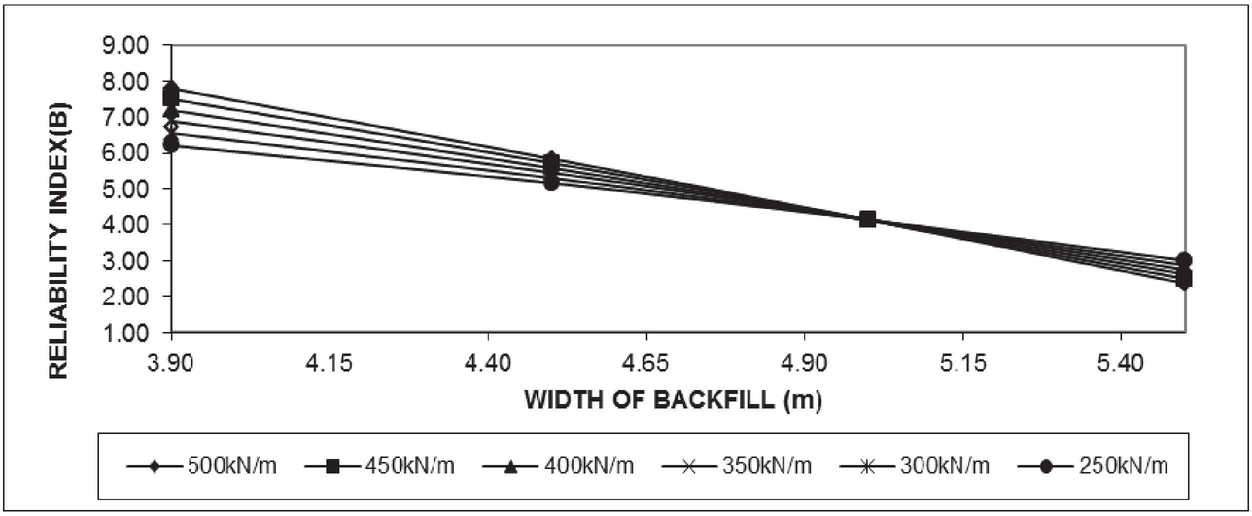

Figure 4: Reliability index versus width of backfill of retaining wall.

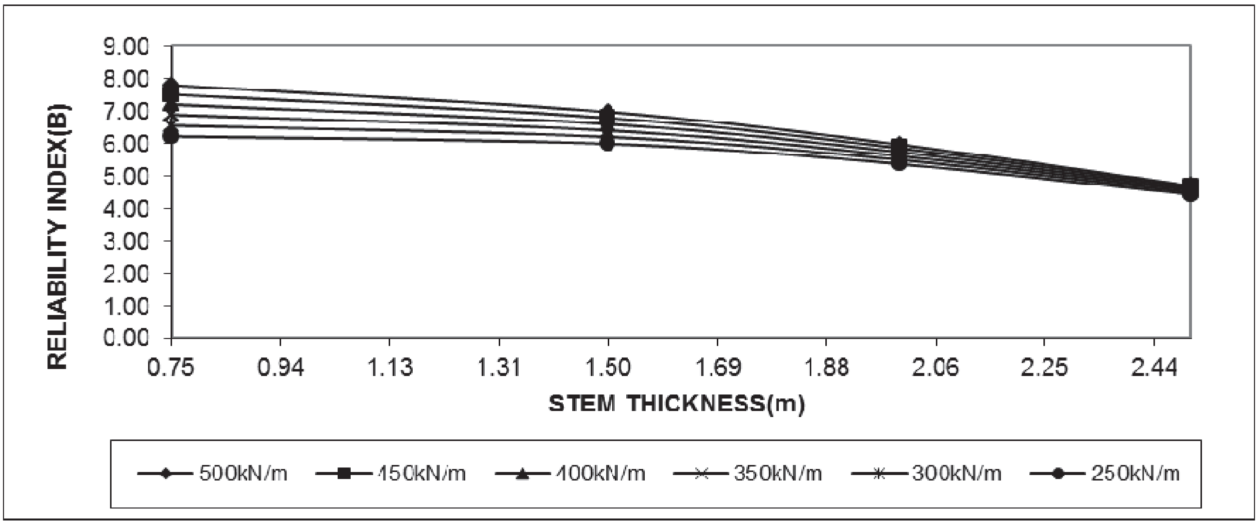

Figure 5: Reliability index versus thickness of stem of retaining wall.

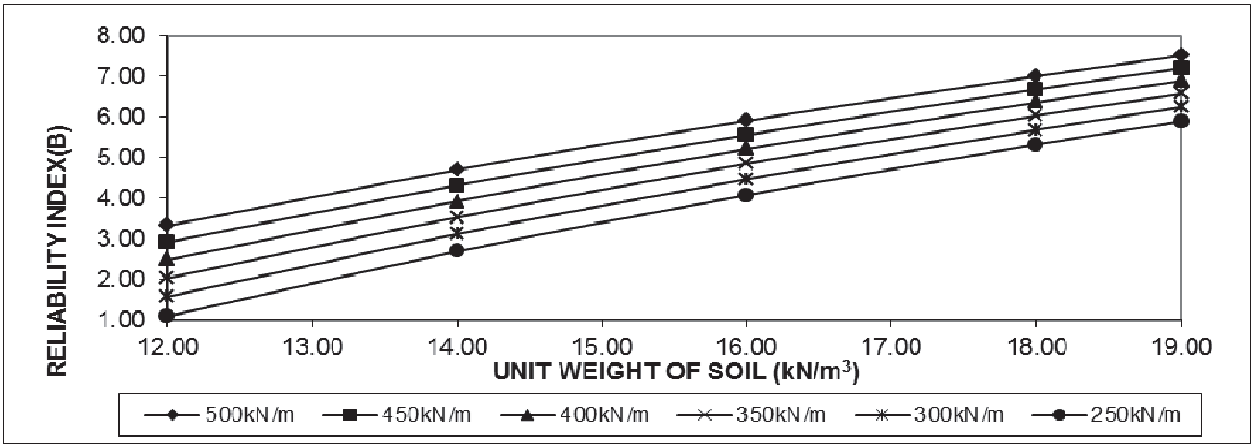

Figure 6: Reliability index versus unit weight of soil of stem of retaining wall.

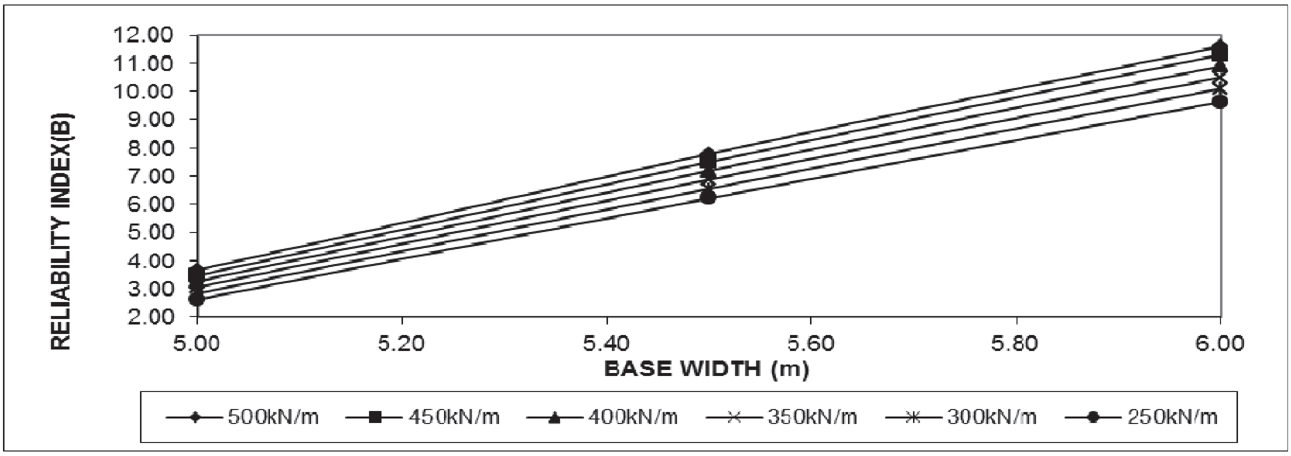

Figure 7: Reliability index versus width of base of retaining wall.

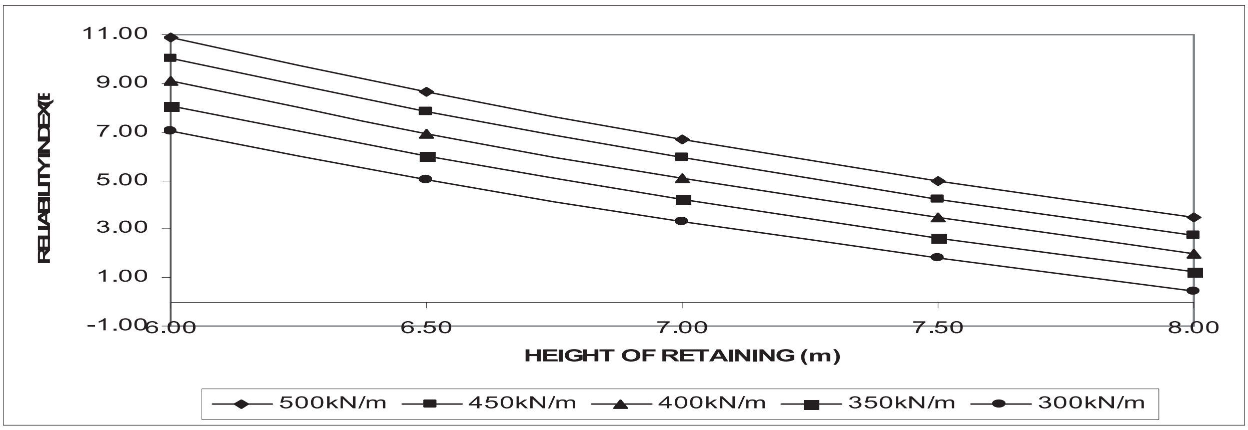

Figure 8: Reliability index versus height of retaining wall.

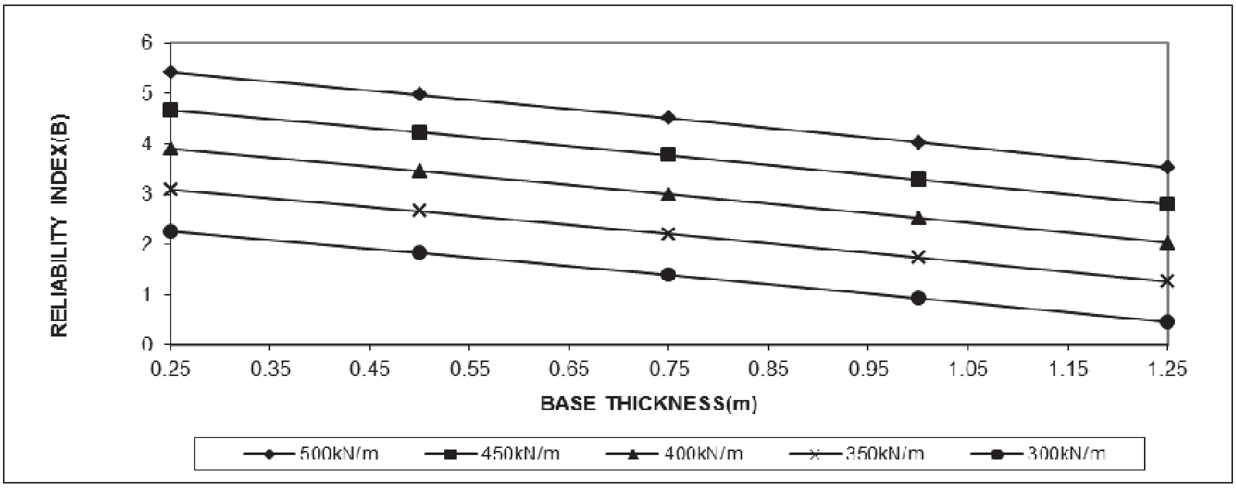

Figure 9: Reliability index versus base thickness of retaining wall.

(20%), the safety of the wall increased by 5.32%. It was observed from figure 2 as the height of the wall was increased by 0.50m (6.25%), the safety of the structure decreased by 32.40%. Similarly from figures 3 and 4, when the base thickness was increased by 0.50m (40%) and the backfill width increased by 0.50m (15.38%), it was observed that the safety decreased by 17.33% and 25.03% respectively.

Also from figure 5, increasing the retaining wall stem thickness by 0.75m (100%) the structural safety decreased by 10.65%. The results from figure 4 indicate that at a width of backfill of 5m the safety level was at its optimum value [14–17], and the convergence means that all the design variables used for the deterministic design are to have the same reliability index.

Again from figure 6, by increasing the magnitude of the unit weight of backfill by 1kN/m^3 (5.56%), the safety of the structure increased by 7.29%, while decreasing the unit weight of backfill by 2kN/m3 (11.11%), the safety of the structure decreased by 15.57%. And from figure 7, it was observed that by increasing the base width by 0.5m, the safety of the structure increased by 52.88%.

3.3.2. Sliding mode of failure

The structure was checked for sliding failure mode and reliability results were estimated for different variables ranging from the height, base slab thickness, width of backfill, stem thickness, weight of soil, and base slab width. The results are as presented graphically in figures 8 to 13.

It was found out that as the magnitude of the design load was increased by 50kN/m^2 (20%), the safety of the structure increased by 6.13%. It was observed from figure 8 that as the height of the wall thickness was increased by 0.50m (6.25%) the safety of the structure decreased by 44.51%. From figure 9, when the base thickness was increased by 0.50m (40%) it was observed that the safety of the structure decreased by 12.41%.

Also from figure 10, when the magnitude of the width of backfill was increased by 0.50m (15.38%) the structural safety increased by 33.43%. It was also observed from figure 11 that when the magnitude of the retaining wall thickness was increased by 0.25m (33.33%), the safety of the structure increased by 24.65%. From figures 12 and 13, as the unit weight of soil and base width of the wall were increased by 2kN/m^3 (10.53%) and 1.00m (18.18%) respectively, the safety of the structure correspondingly increased by 25% and 29%.

3.3.3. Bearing capacity mode of failure

The structure was checked for bearing capacity failure mode and reliability results were estimated for different variables ranging from the height, base slab thickness, width of backfill, stem thickness, weight of soil, and base slab width. The results are as presented graphically in figures 14 to 18.

It was found out that when the magnitude of the design loads on the retaining wall was increased by 50kN/m^2 (20%) the safety of the structure increased by 25.53%. It was observed from figure 14 that as the height of the wall was increased by 0.01m (0.16%), the safety of the structure increased by 0.83%. Again from figure 15, when the width of back fill was increased by 0.5m (15.38%) it was observed that the safety of the structure decreased by 34.32%.

It was also observed from figure 16 that by increasing the value of the bearing capacity factor N_q by 2 (1.96%) the safety of the structure increased by 3.49%. However from figure 17, increasing the value of the bearing capacity factor N_c by 5 (4.54%) the safety of the structure increased by 7.46%. And from figure 18, by increasing the value of N_γ by 1 (2.22%) the safety of the structure increased by 4.05%.

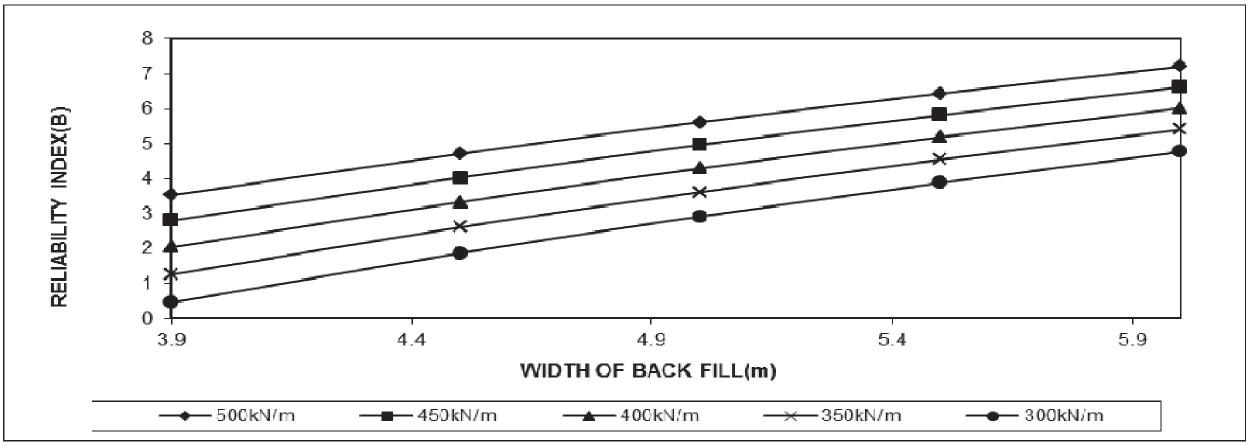

Figure 10: Reliability index versus width of backfill of retaining wall.

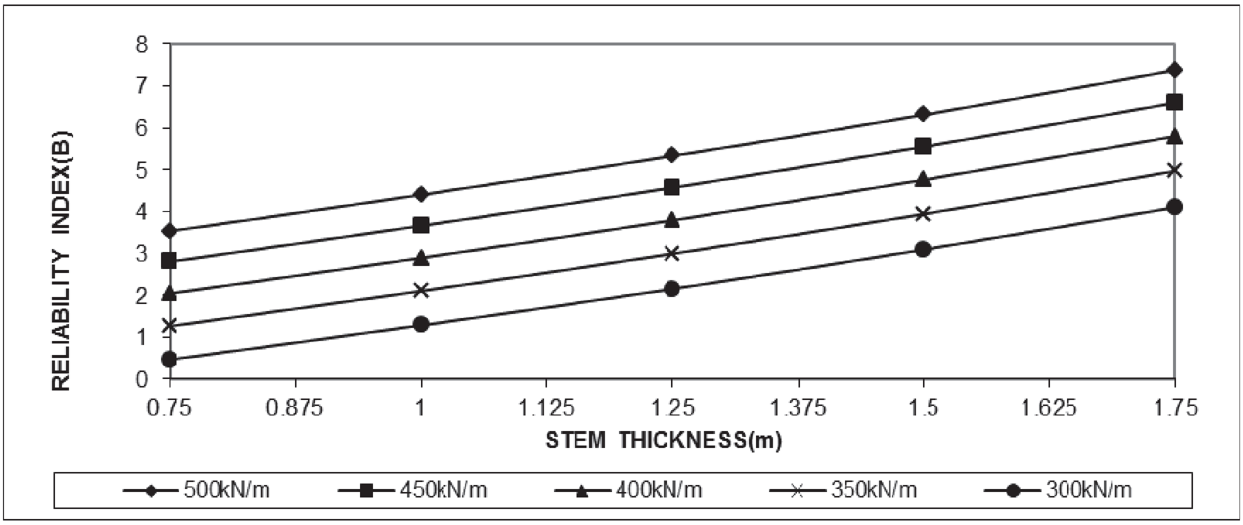

Figure 11: Reliability index versus thickness of stem of retaining wall.

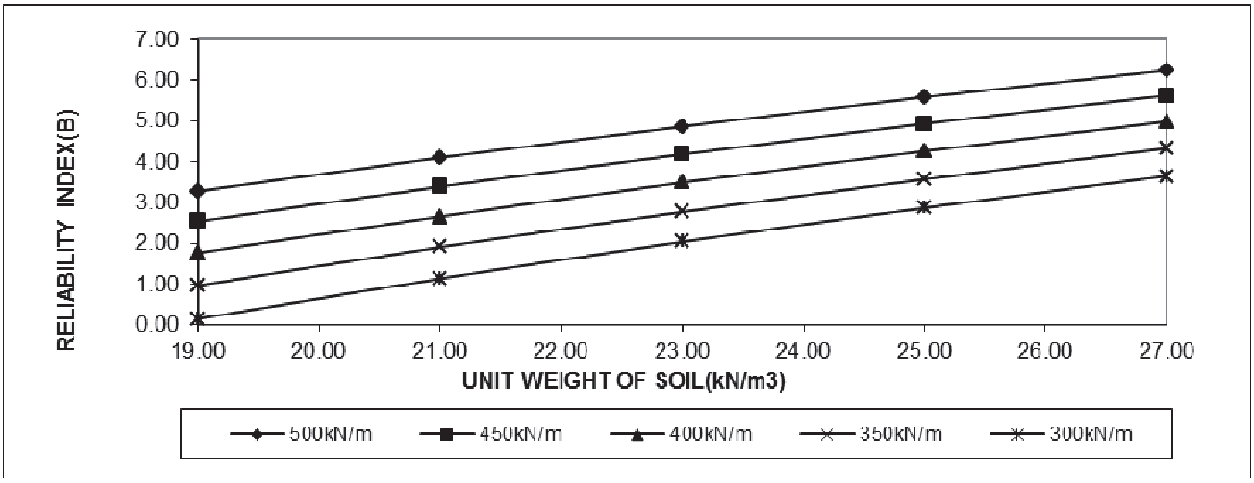

Figure 12: Reliability index versus unit weight of soil of stem of retaining wall.

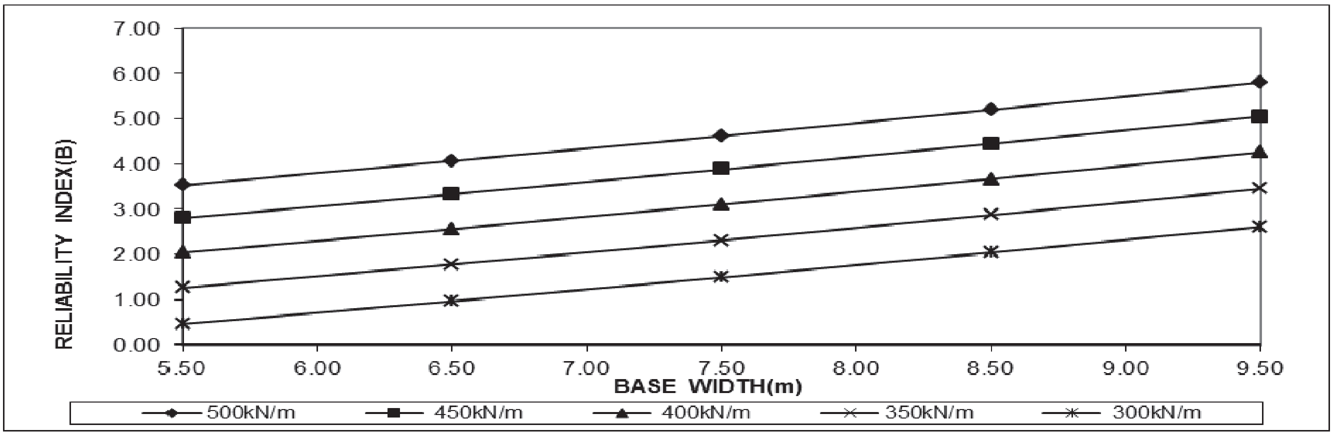

Figure 13: Reliability index versus base width of retaining wall.

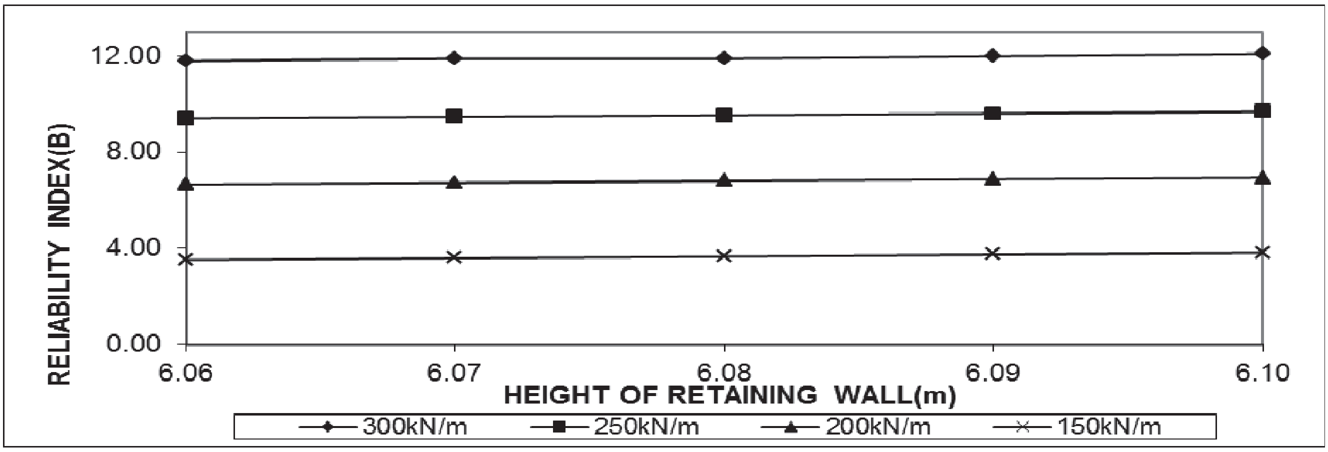

Figure 14: Reliability index versus height of retaining wall.

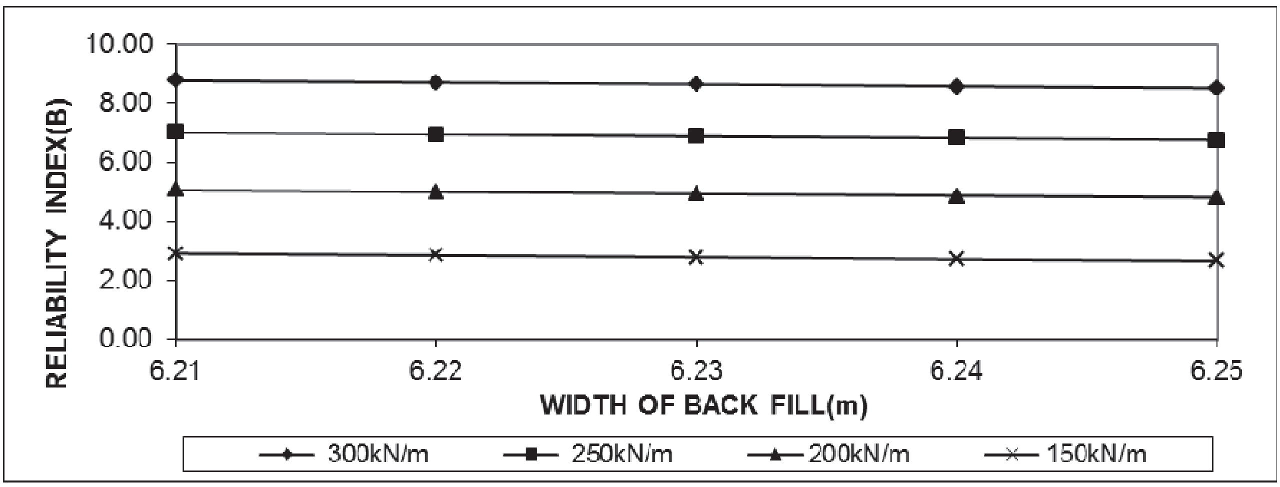

Figure 15: Reliability index versus width of backfill of retaining wall.

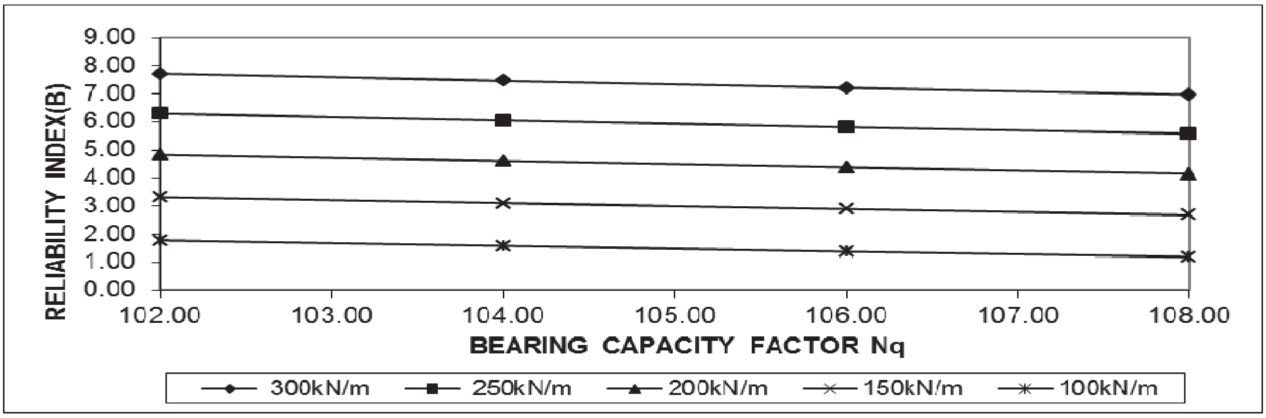

Figure 16: Reliability index versus bearing capacity factor, N_q.

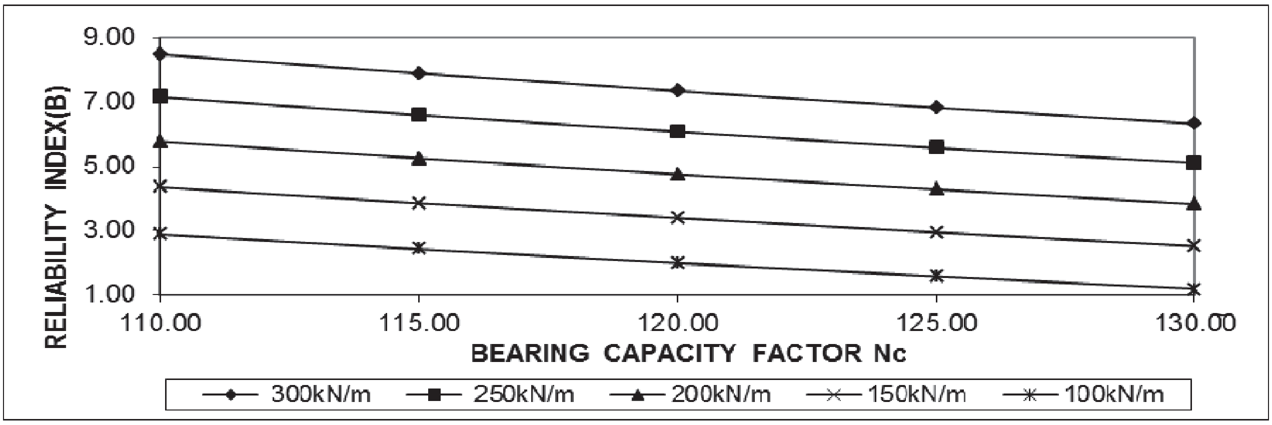

Figure 17: Reliability index versus bearing capacity factor, N_c.

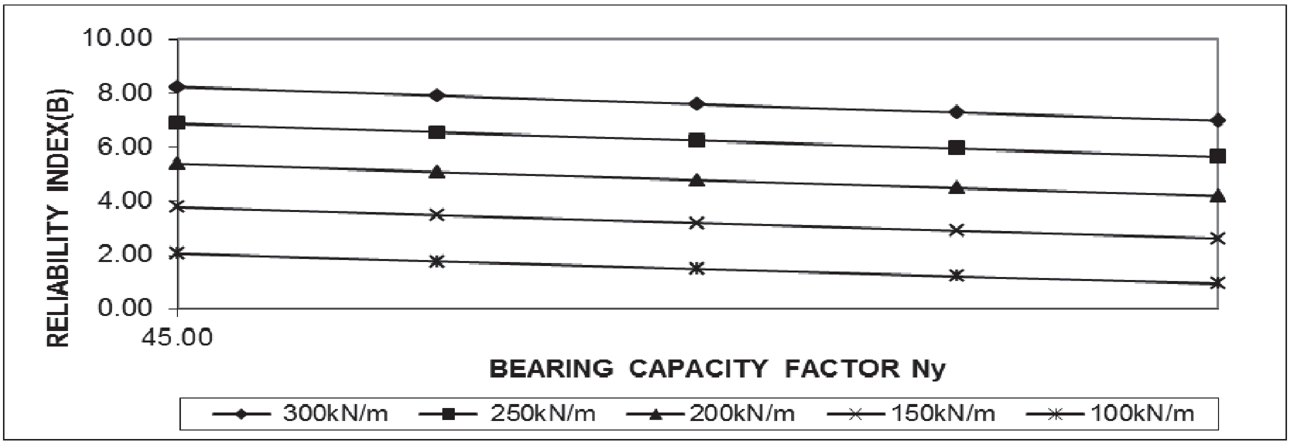

Figure 18: Reliability index versus bearing capacity factor, N_γ.

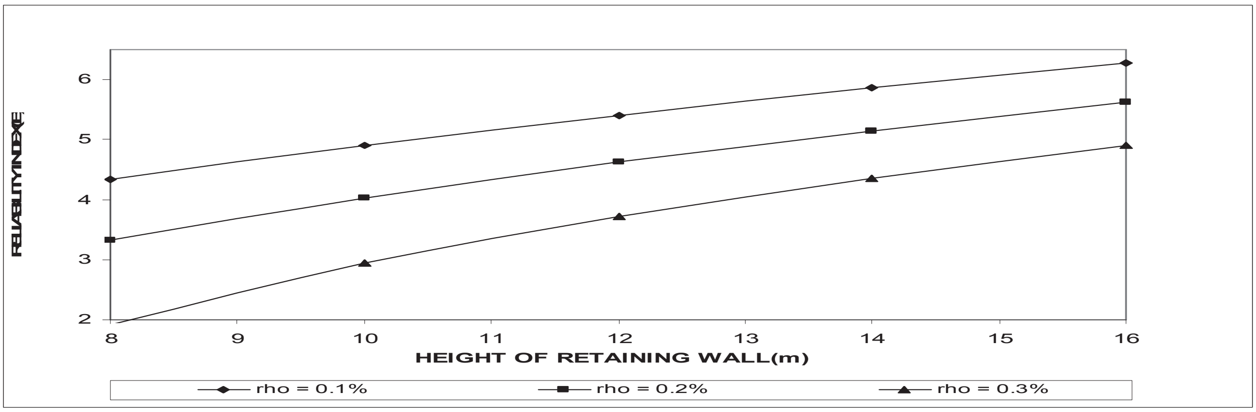

Figure 19: Reliability index versus height of retaining wall.

3.3.4. Bending moment failure mode

The structure was checked for bending moment failure mode and reliability results were estimated for different variables ranging from the height, base slab thickness, width of backfill, stem thickness, weight of soil, and base slab width. The results are as presented graphically in figures 19 to 22.

It was found out that when the magnitude of the reinforcement ratio of steel was increased by 0.10%, the safety of the structure increased by 18.43%. From figure 19 when the height of the wall was increased by 2m (25%) the safety of the structure increased by 13.16%. Also, from figure 20, by increasing the width of backfill by 0.5m (15.38%) it was observed that the structural safety increased by 4.12%. From figure 21, increasing the base slab thickness by 0.10m (12.5%) the safety decreased by 0.87%, and by increasing the width of the base slab (from figure 22) by 1m (10%) the safety of the structure increased by 2.26%.

4. Conclusion

Failure investigation of a cantilever retaining wall considering the design requirements of BS8110 [7] and BS8004 [8] was presented. The failure criteria of overturning, sliding, bearing capacity and bending were examined using FORM in estimating the reliability indices when all the variables regarding loading, geometry and material properties were considered as random.

It was shown that as the magnitude of the design load was increased by 50kN/m^2, the safety of the retaining wall increased by about 5%, 6% and 25% considering overturning, sliding and bearing capacity failure modes respectively. It was also observed that as the magnitude of reinforcement ratio was increased by 0.1%, the safety of the structure increased by about 18%. Also, at a width of backfill of 5m the safety level was at its optimum value considering overturning failure mode and is in agreement with JCSS [14].

Again, when the width of backfill was increased by about 15%, the safety of the structure decreased by about 25% and 34% considering overturning and bearing capacity failure modes; and increased by about 33% and 4% considering sliding, and bending moment failure modes respectively.

Based on the reliability indices obtained, the bending moment mode of failure was the most critical, followed by the sliding, the overturning, and by the bearing capacity modes of failure.

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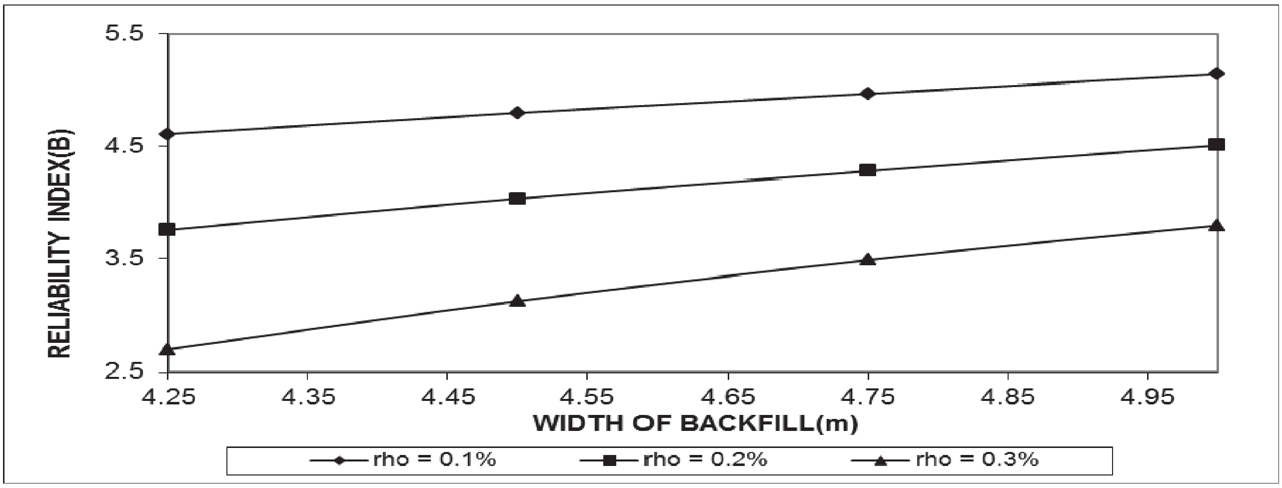

Figure 20: Reliability index versus width of backfill of retaining wall.

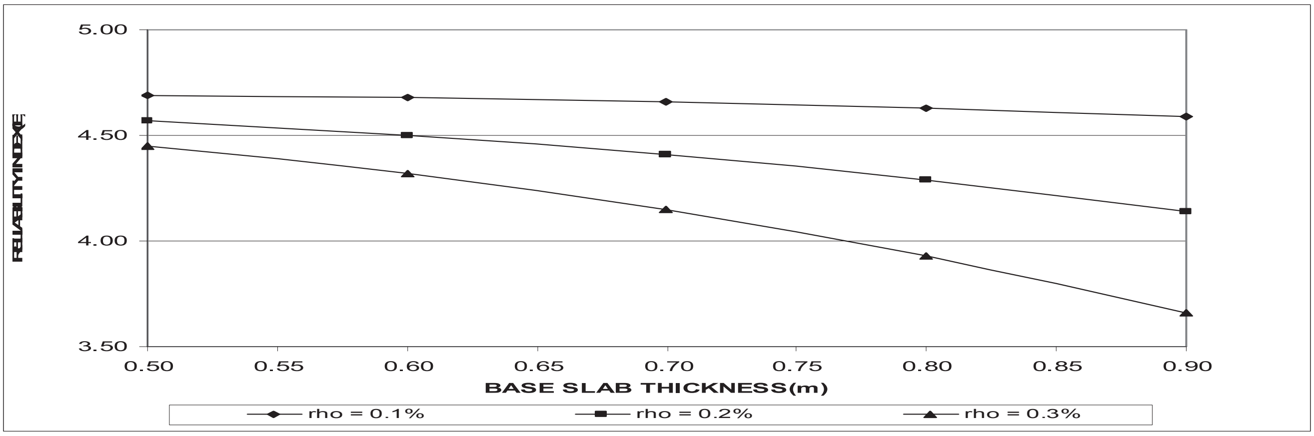

Figure 21: Reliability index versus base slab thickness of retaining wall.

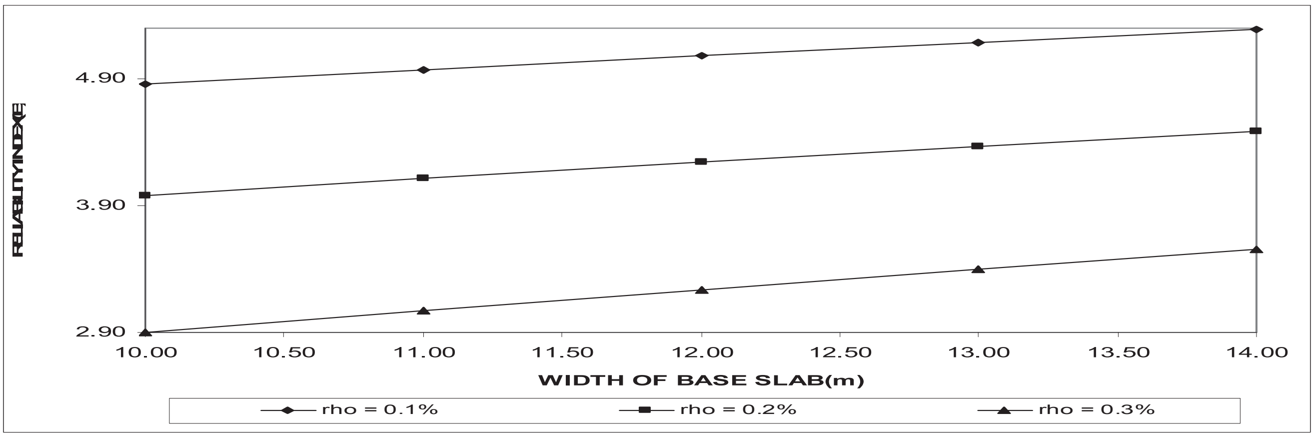

Figure 22: Reliability index versus width of base slab of retaining wall.

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